Trigonometric Functions - Answers

June 2017 Mathematics Advanced Paper 1: Pure Mathematics 3

1.

Question Number	Scheme	Marks
4.(a)	$R=\sqrt{29}$	B1
	$\tan\alpha=\frac{2}{5}\Rightarrow\alpha=$ awrt 0.381	M1A1
		(3)
(b)	$5\cot 2x-3\operatorname{cosec}2x=2\Rightarrow 5\frac{\cos 2x}{\sin 2x}-\frac{3}{\sin 2x}=2$	M1
	$\Rightarrow 5\cos 2x-2\sin 2x=3$	A1
		(2)
(c)	$5\cos 2x-2\sin 2x=3\Rightarrow\cos(2x+0.381)=\frac{3}{\sqrt{29}}$	M1
	$2x+0.381=\arccos\left(\frac{3}{\sqrt{29}}\right)\Rightarrow x=...$	dM1
	$x=$ awrt 0.30, 2.46	A1A1
		(4)
		(9 marks)

Alt I (c)	$5\cos 2x-2\sin 2x=3\Rightarrow 10\cos^2 x-5-4\sin x\cos x=3$	
	$\Rightarrow 4\tan^2 x+2\tan x-1=0$	M1
	$\Rightarrow\tan x=\frac{-1\pm\sqrt{5}}{4}\Rightarrow x=..$	dM1
	$x=$ awrt 0.30, 2.46	A1A1
		(4)
Alt II (c)	$5\cos 2x-2\sin 2x=3\Rightarrow(5\cos 2x)^2=(3+2\sin 2x)^2$ & $\cos^2 2x=1-\sin^2 2x$	
	$\Rightarrow 29\sin^2 2x+12\sin 2x-16=0$	M1
	$\Rightarrow\sin 2x=\frac{-12\pm\sqrt{2000}}{58}\Rightarrow 2x=..\Rightarrow x=..$	dM1
	$x=$ awrt 0.30, 2.46	A1A1
		(4)

(a)

B1 $R=\sqrt{29}$

Condone $R=\pm\sqrt{29}$ (Do not allow decimals for this mark Eg 5.39 but remember to isw after $\sqrt{29}$)

M1 $\tan\alpha=\pm\frac{2}{5}, \tan\alpha=\pm\frac{5}{2}\Rightarrow\alpha=...$

If R is used to find α accept $\sin\alpha=\pm\frac{2}{R}$ or $\cos\alpha=\pm\frac{5}{R}\Rightarrow\alpha=...$

A1 $\alpha=$awrt 0.381 Note that the degree equivalent $\alpha=$awrt $21.8°$ is A0

(b)

M1 Replaces $\cot 2x$ by $\frac{\cos 2x}{\sin 2x}$ **and** $\operatorname{cosec} 2x$ by $\frac{1}{\sin 2x}$ in the lhs

Do not be concerned by the coefficients 5 and -3.

Replacing $\cot 2x$ by $\frac{1}{\tan 2x}$ does not score marks until the $\tan 2x$ has been replaced by $\frac{\sin 2x}{\cos 2x}$

They may state $\times\sin 2x\Rightarrow 5\cos 2x-3=2\sin 2x$ which implies this mark

A1 cso $5\cos 2x-2\sin 2x=3$ There is no need to state the value of 'c'

The notation must be correct. They cannot mix variables within their equation

Do not accept for the final A1 $\tan 2x=\frac{\sin}{\cos}2x$ within their equations

(c)

M1 Attempts to use part (a) and (b). They must be using their R and α from part (a) and their c from part (b)

Accept $\cos(2x\pm'\alpha')=\frac{'c'}{'R'}$ Condone $\cos(\theta\pm'\alpha')=\frac{'c'}{'R'}$ or even $\cos(x\pm'\alpha')=\frac{'c'}{'R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach $x = ..$
Don't be concerned if they change the variable in the question and solve for $\theta =$ (as long as all operations have been undone).You may not see any working. It is implied by one correct answer.
You may need to check with a calculator.

Eg for an incorrect α $\cos(2x+1.19) = \frac{3}{\sqrt{29}} \Rightarrow x = -0.105$ would score M1 dM1 A0 A0

A1 One solution correct, usually $x = 0.3 / 0.30$ or $x = 2.46$ or in degrees $17.2°$ **or** $141.(0)°$

A1 Both solutions correct awrt $x =$ awrt $0.30, 2.46$ and no extra values in the range.
Condone candidates who write 0.3 and 2.46 without any (more accurate) answers
In degrees accept awrt 1 dp $17.2°, 141.(0)°$ and no extra values in the range.

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Special case: For candidates who are misreading the question and using their part (a) with 2 on the rhs.
They will be allowed to score a maximum of SC M1 dM1 A0 A0

M1 Attempts to use part (a) with 2. They must be using their R and α from part (a)

Accept $\cos(2x \pm '\alpha') = \frac{2}{'R'}$ Condone $\cos(\theta \pm '\alpha') = \frac{2}{'R'}$ or even $\cos(x \pm '\alpha') = \frac{2}{'R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach $x = ..$
You may not see any working. It is implied by one correct answer. You may need to check with a calculator.

Eg for an correct α and R $\cos(2x+0.381) = \frac{2}{\sqrt{29}} \Rightarrow x = 0.405$

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Alt to part (c)

M1 Attempts both double angle formulae condoning sign slips on cos2x , divides by $\cos^2 x$
and forms a quadratic in tan by using the identity $\pm 1 \pm \tan^2 x = \sec^2 x$

dM1 Attempts to solve their quadratic in tan$x$ leading to a solution for x.

A1 A1 As above

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2.

Question	Scheme	Marks
3.(a)	$R=\sqrt{5}$	B1
	$\tan\alpha=\frac{1}{2}\Rightarrow\alpha=26.57°$	M1A1
		(3)
(b)	$\frac{2}{2\cos\theta-\sin\theta-1}=15\Rightarrow\frac{2}{\sqrt{5}\cos(\theta+26.6°)-1}=15$	
	$\Rightarrow\cos(\theta+26.6°)=\frac{17}{15\sqrt{5}}=(awrt\ 0.507)$	M1A1
	$\theta+26.57°=59.54°$	
	$\Rightarrow\theta=awrt\ 33.0°$ or $awrt\ 273.9°$	A1
	$\theta+26.6°=360°-$ their '59.5°'	dM1
	$\Rightarrow\theta=awrt\ 273.9°$ and $awrt\ 33.0°$	A1
		(5)
(c)	$\theta-$ their $26.57°=$ their $59.54°\Rightarrow\theta=...$	M1
	$\theta=$ awrt $86.1°$	A1
		(2)
		(10 marks)

(a)

B1 $R=\sqrt{5}$. Condone $R=\pm\sqrt{5}$ Ignore decimals

M1 $\tan\alpha=\pm\frac{1}{2},\ \tan\alpha=\pm\frac{2}{1}\Rightarrow\alpha=...$

If their value of R is used to find the value of α only accept $\cos\alpha=\pm\frac{2}{R}$ OR $\sin\alpha=\pm\frac{1}{R}\Rightarrow\alpha=...$

A1 $\alpha=$ awrt $26.57°$

(b)

M1 Attempts to use part (a) $\Rightarrow\cos(\theta\pm\text{their } 26.6°)=K,\ \ |K|\,,,\,1$

A1 $\cos(\theta\pm\text{their } 26.6°)=\frac{17}{15\sqrt{5}}=(\text{awrt } 0.507)$. Can be implied by $(\theta\pm\text{their } 26.6°)=\text{awrt } 59.5°/59.6°$

A1 One solution correct, $\theta=awrt\ 33.0°$ or $\theta=awrt\ 273.9°$ Do not accept 33 for 33.0.

dM1 Obtains a second solution in the range. It is dependent upon having scored the previous M.
Usually for $\theta\pm\text{their } 26.6°=360°-\text{their } 59.5°\Rightarrow\theta=...$

A1 Both solutions $\theta=awrt\ 33.0°$ and $awrt\ 273.9°$. Do not accept 33 for 33.0.
Extra solutions inside the range withhold this A1. Ignore solutions outside the range $0\,,,\,\theta<360°$

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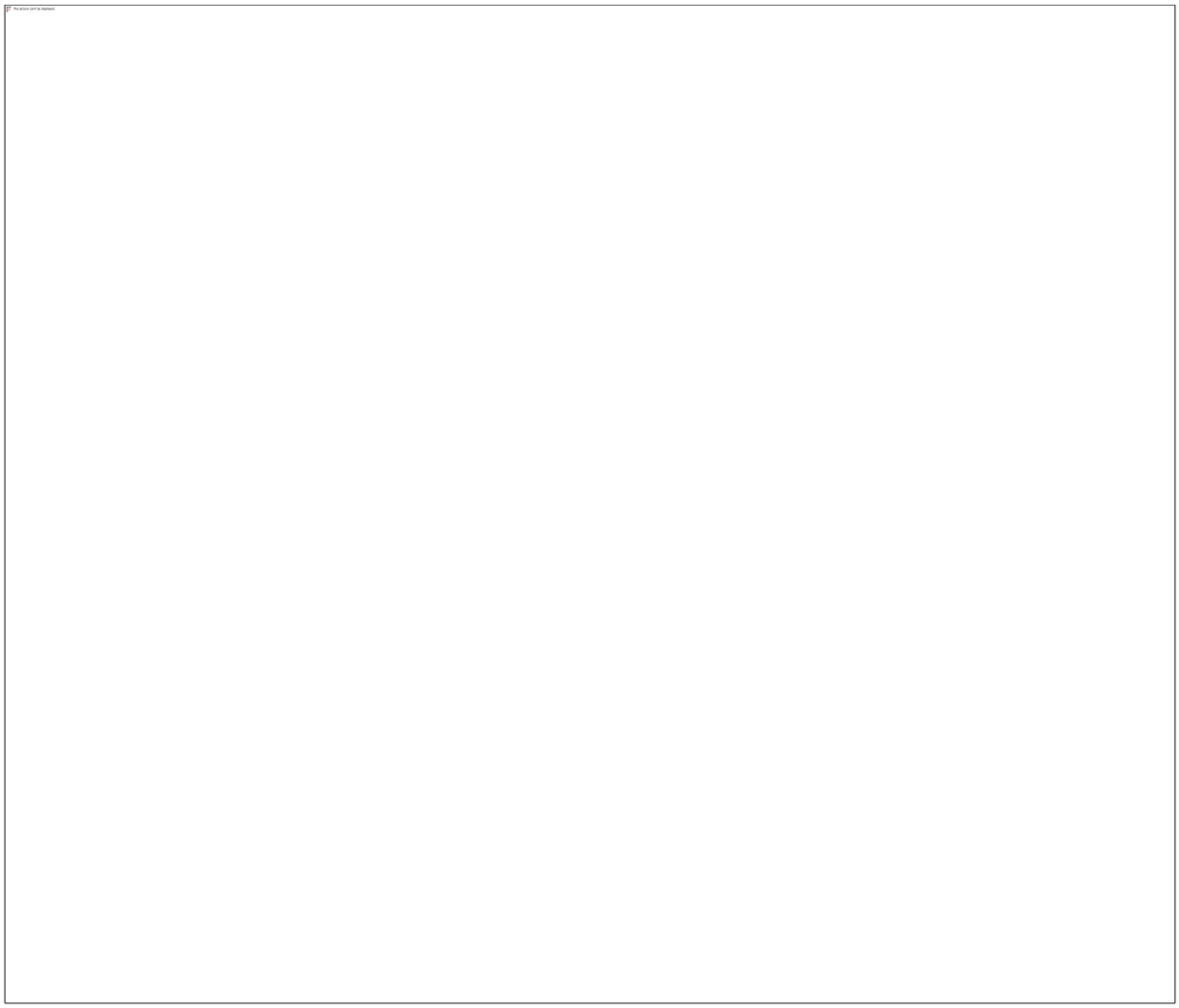
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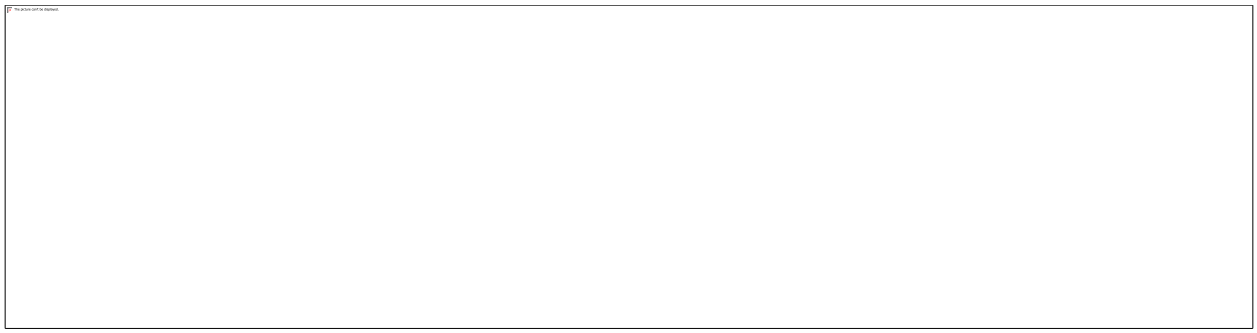

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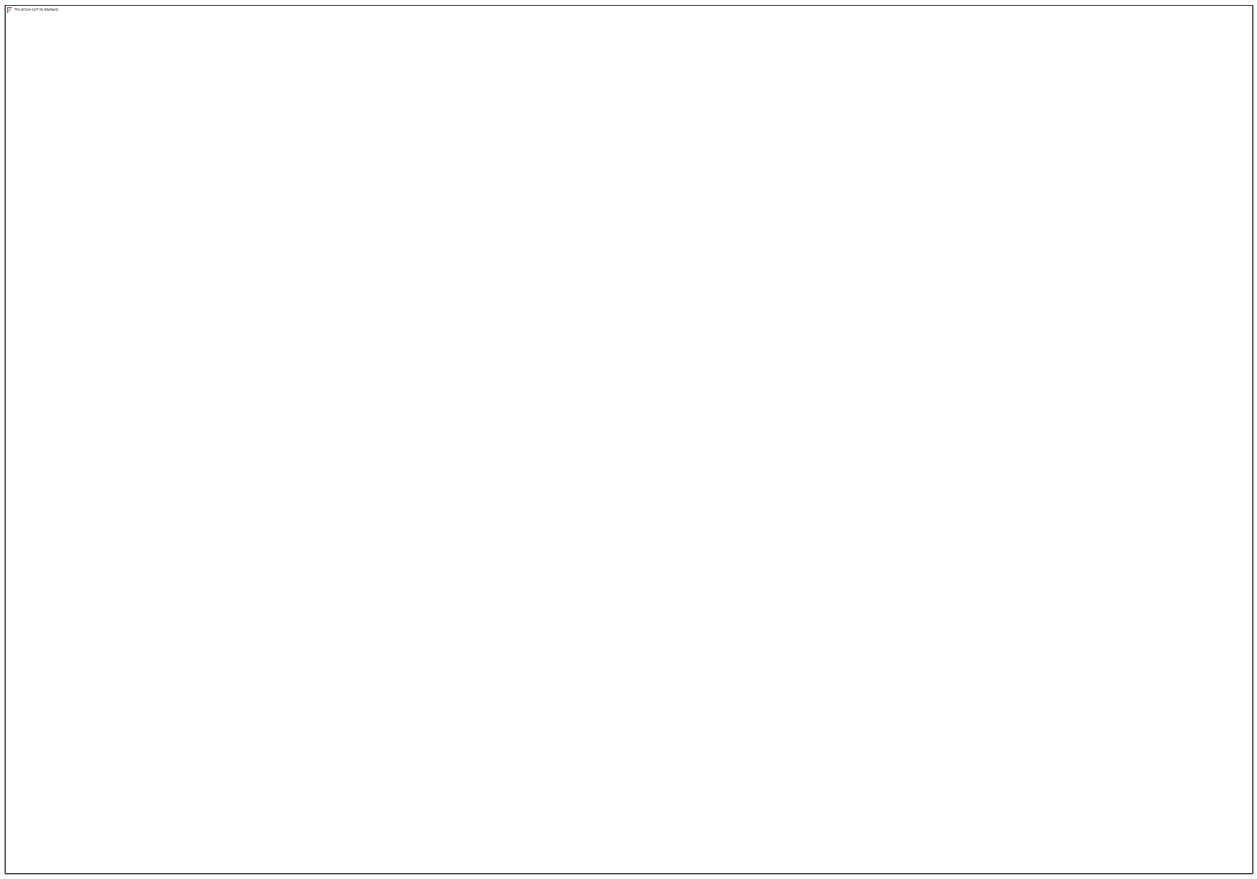
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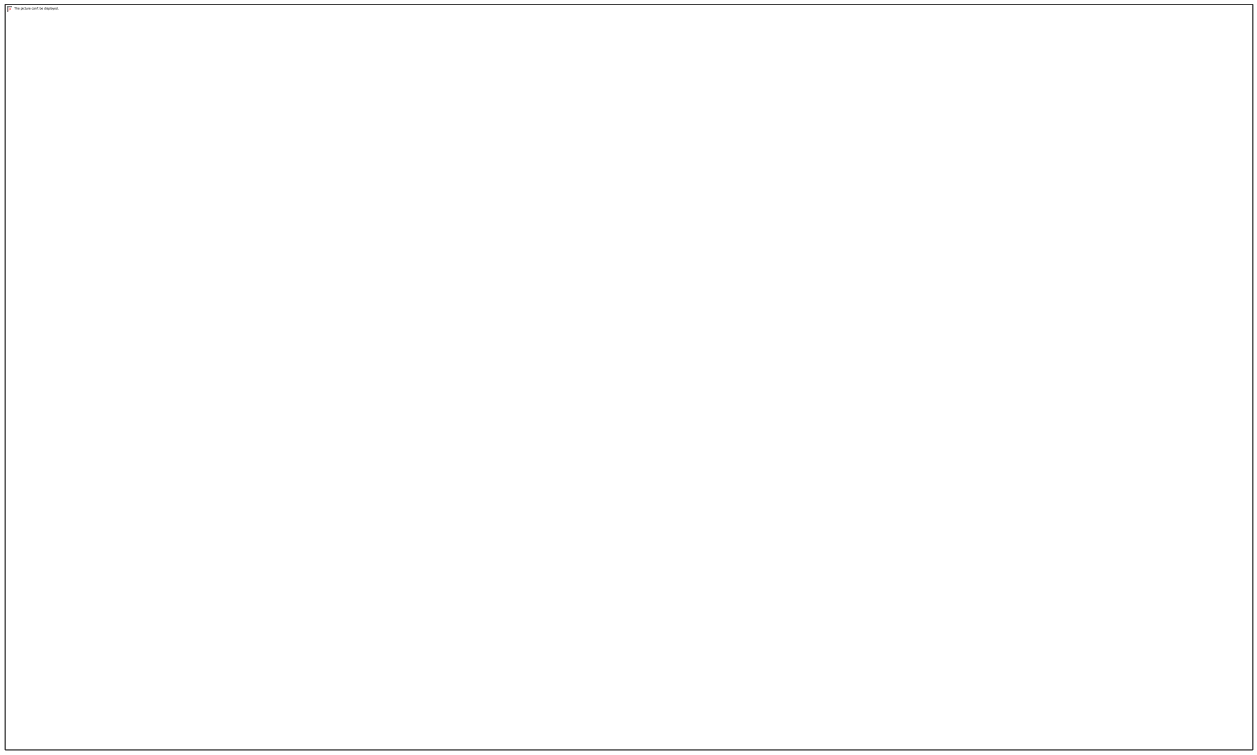
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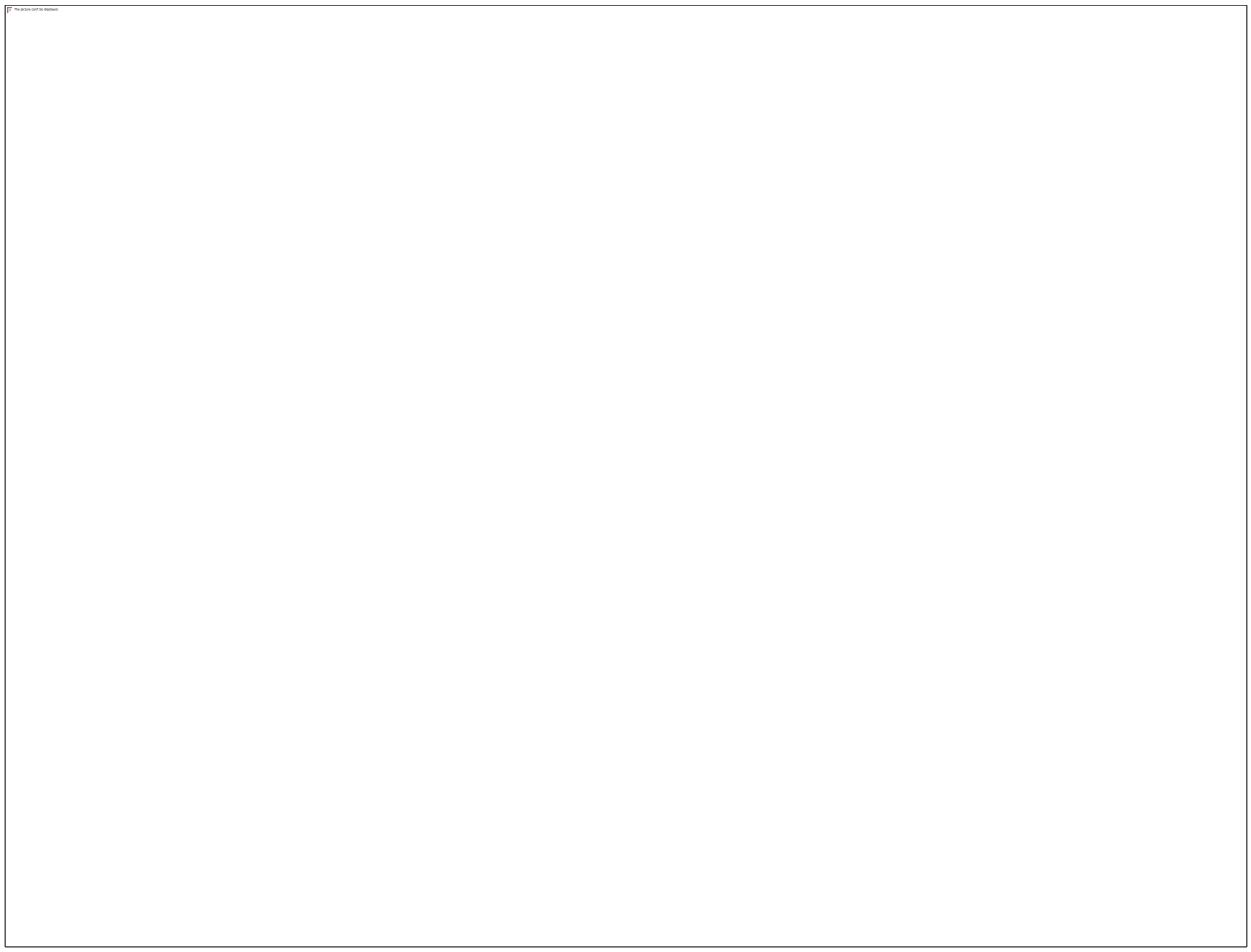
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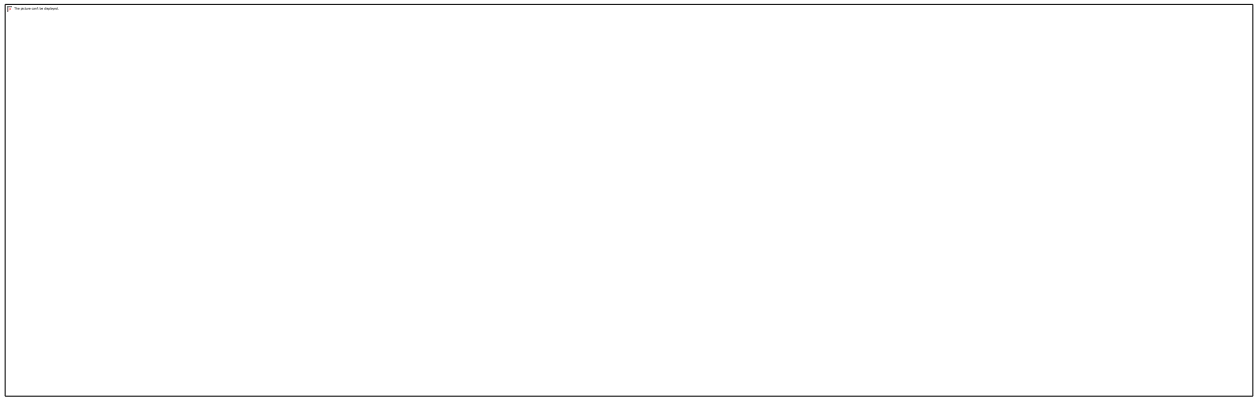
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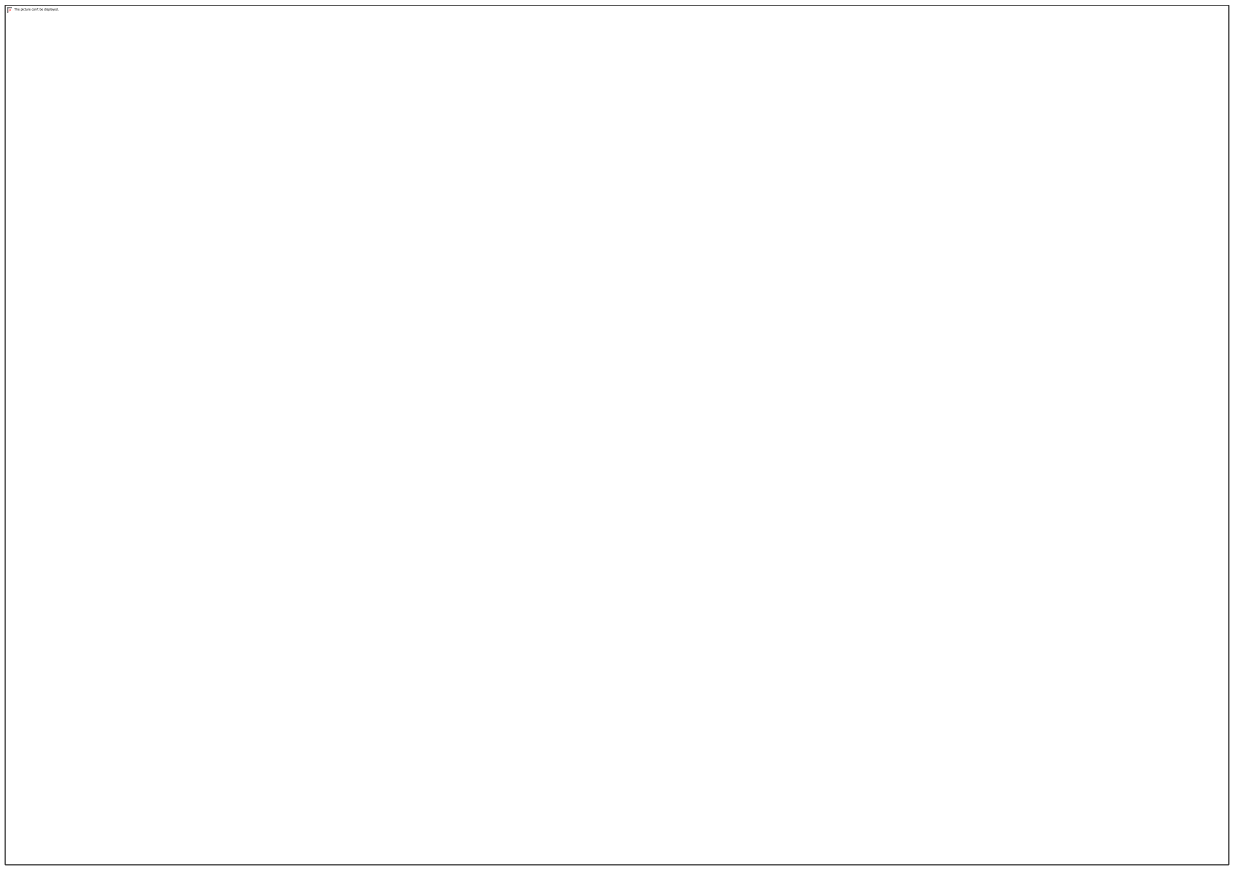
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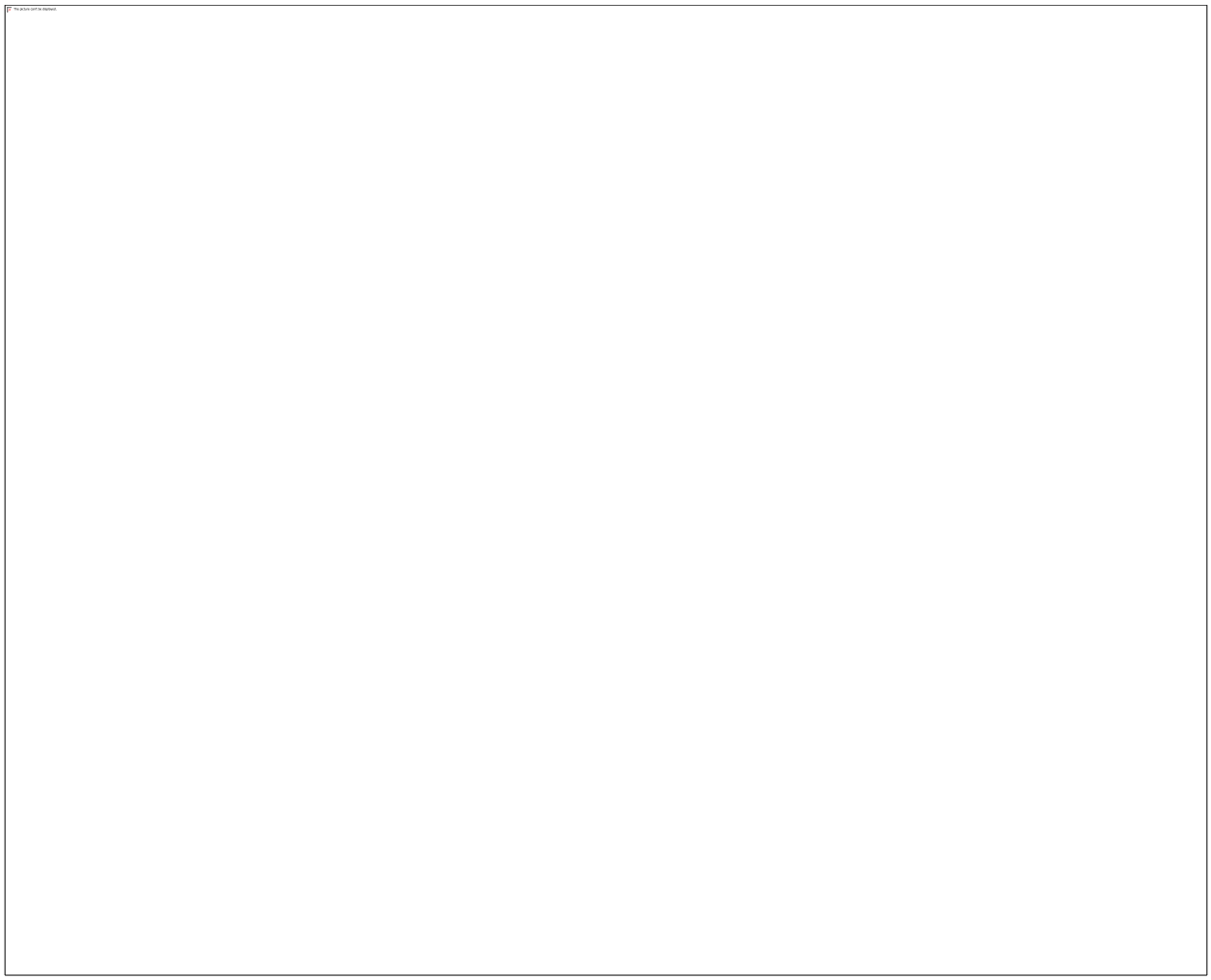

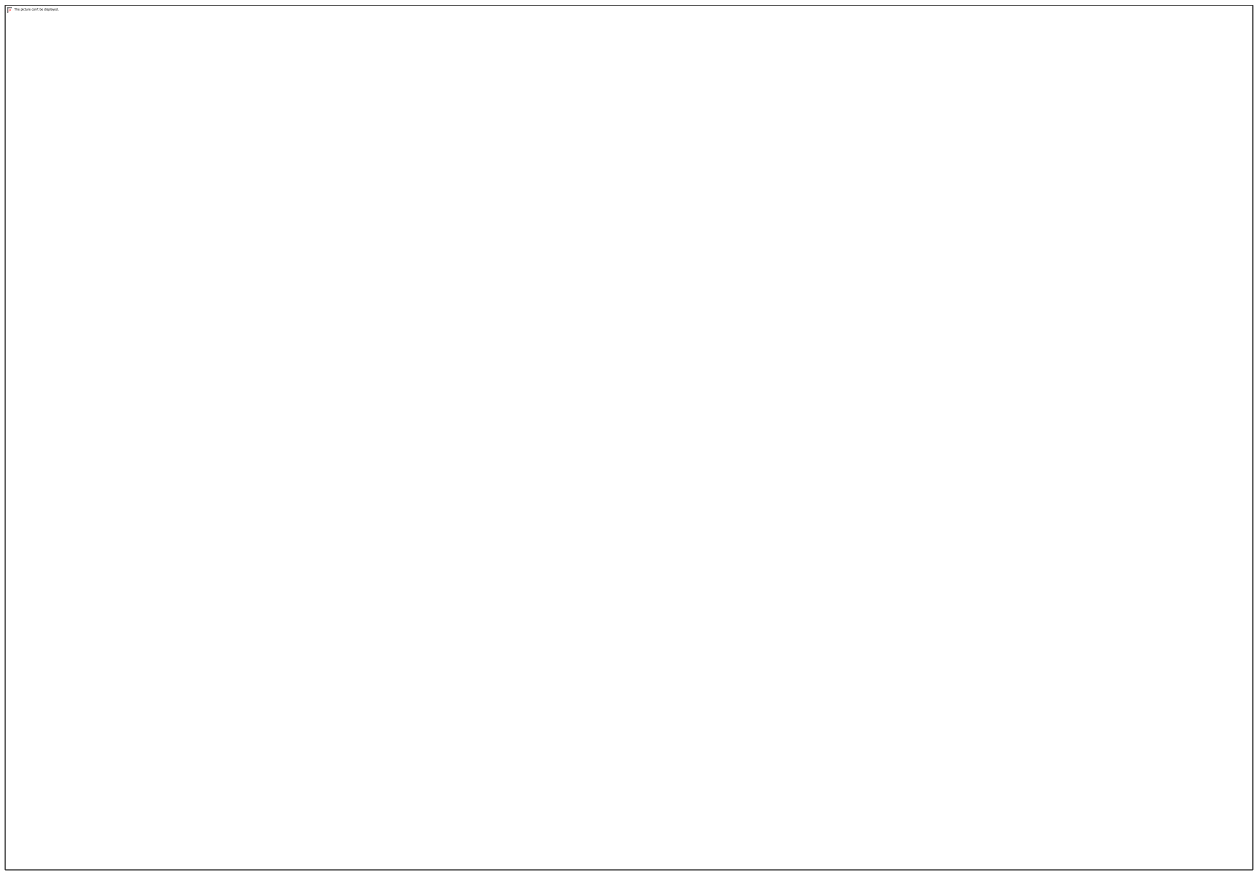
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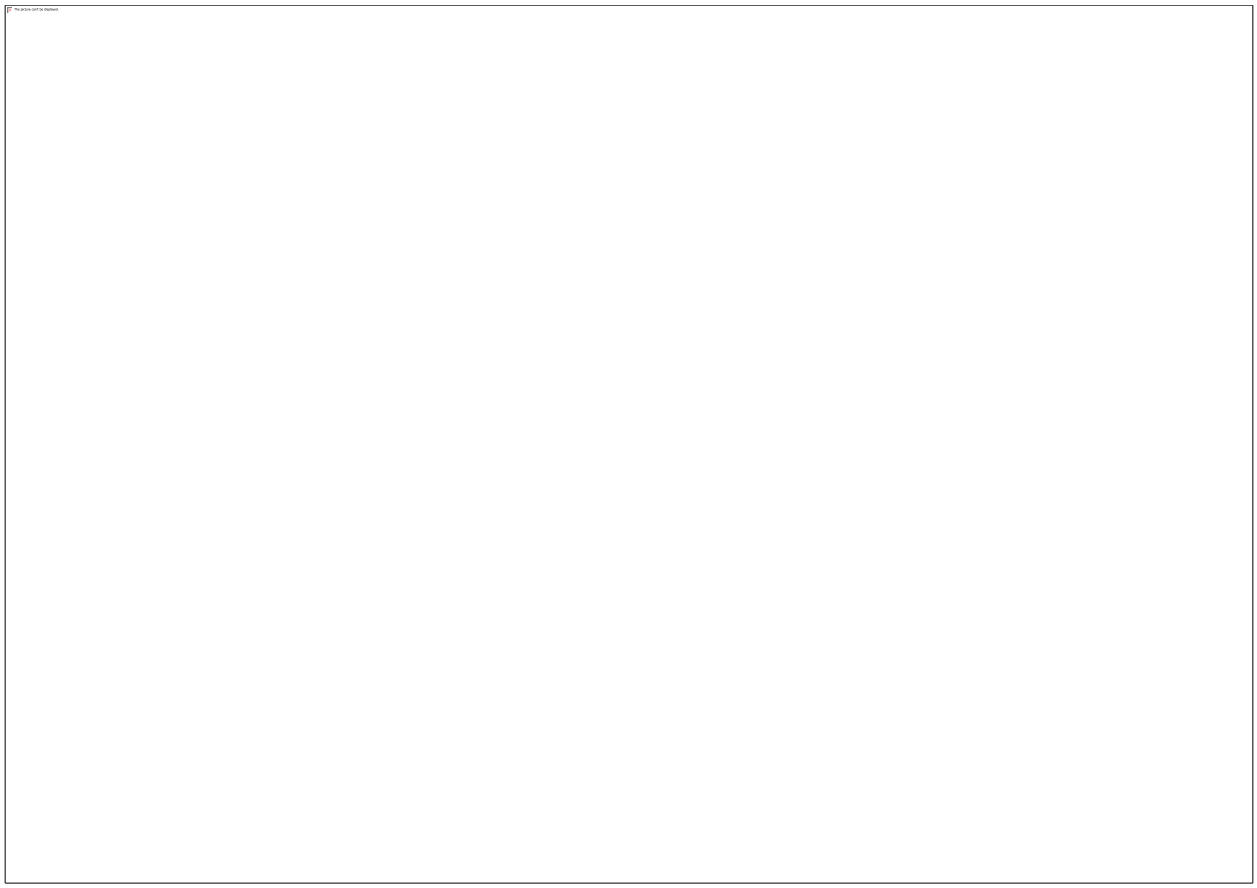
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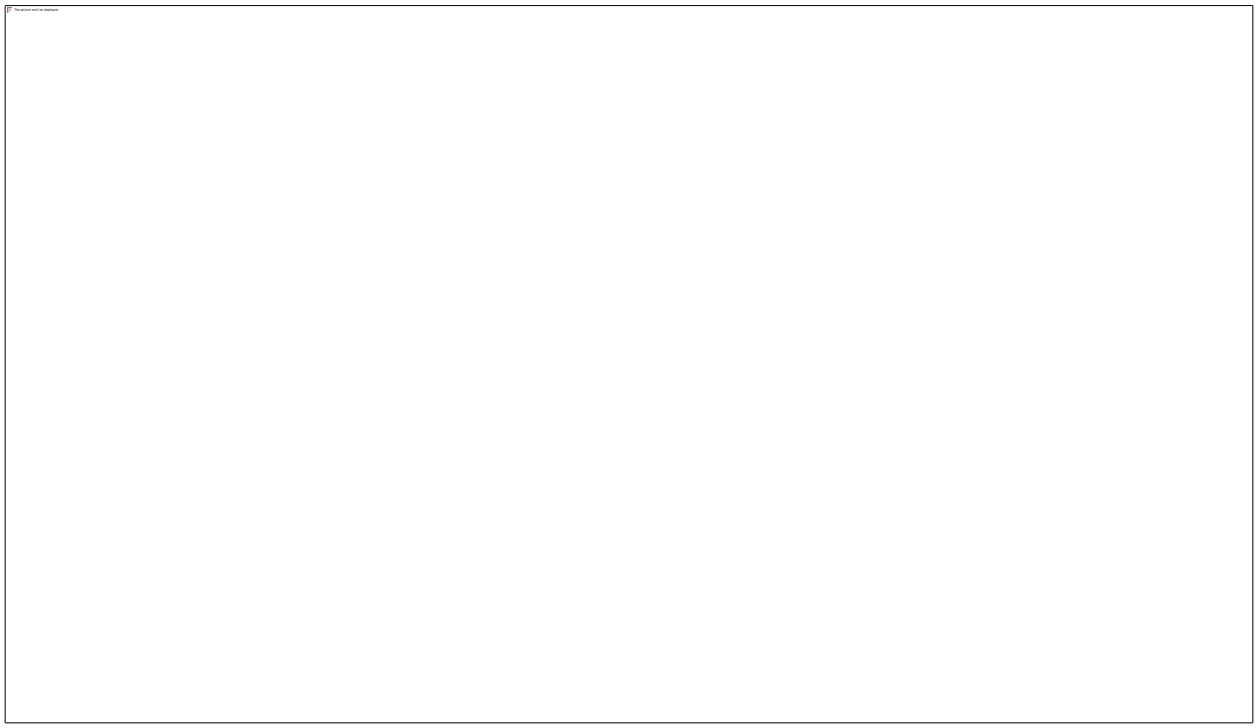
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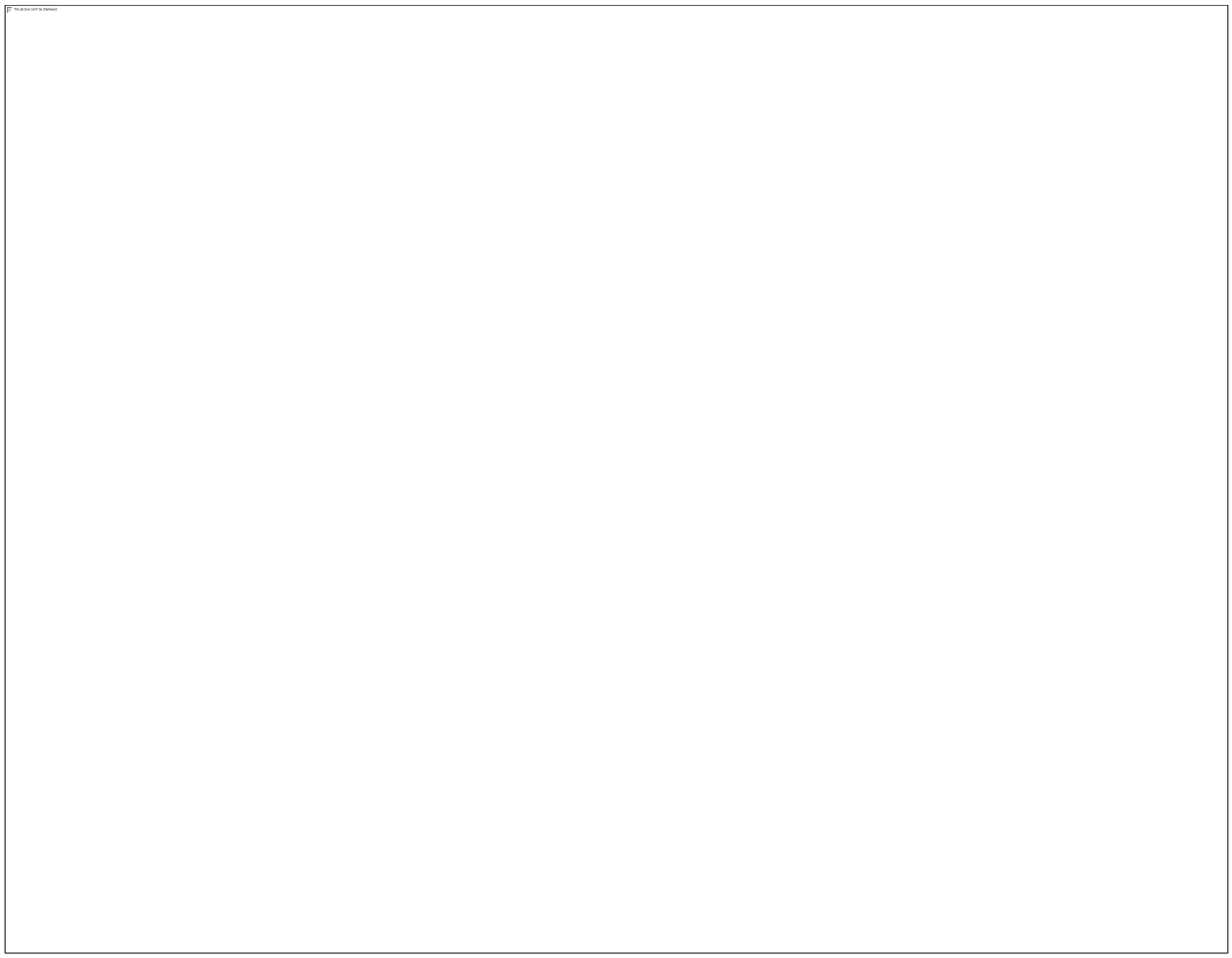
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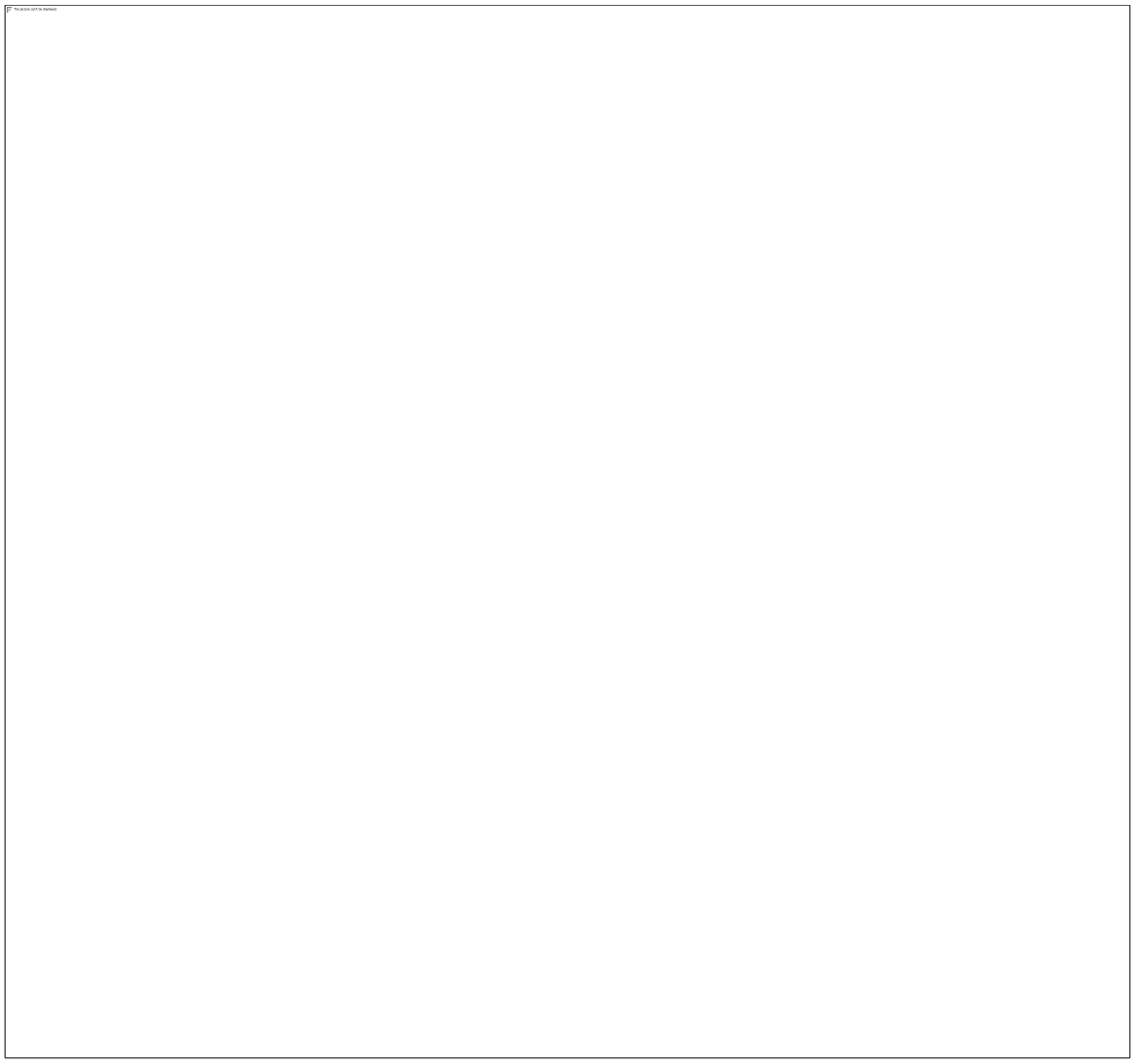
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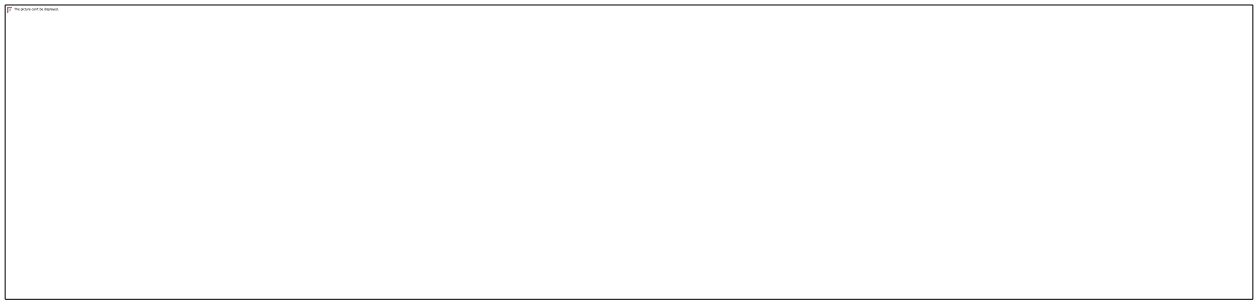
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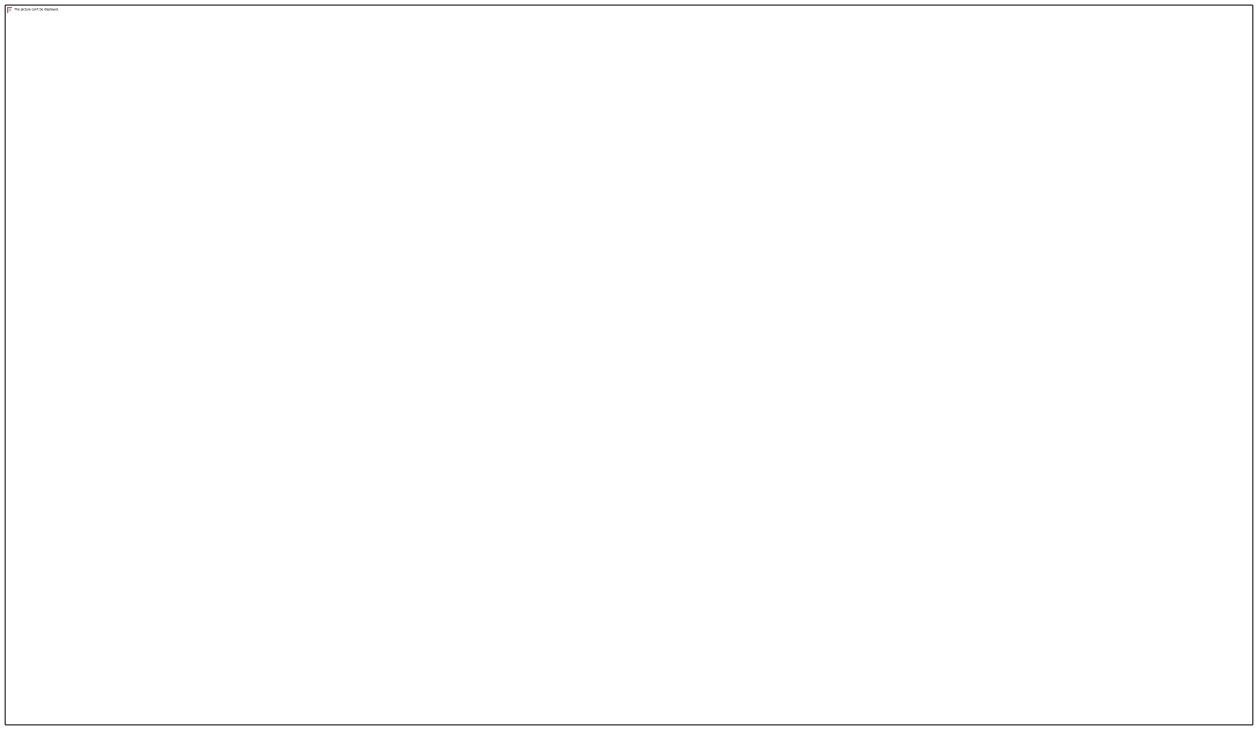

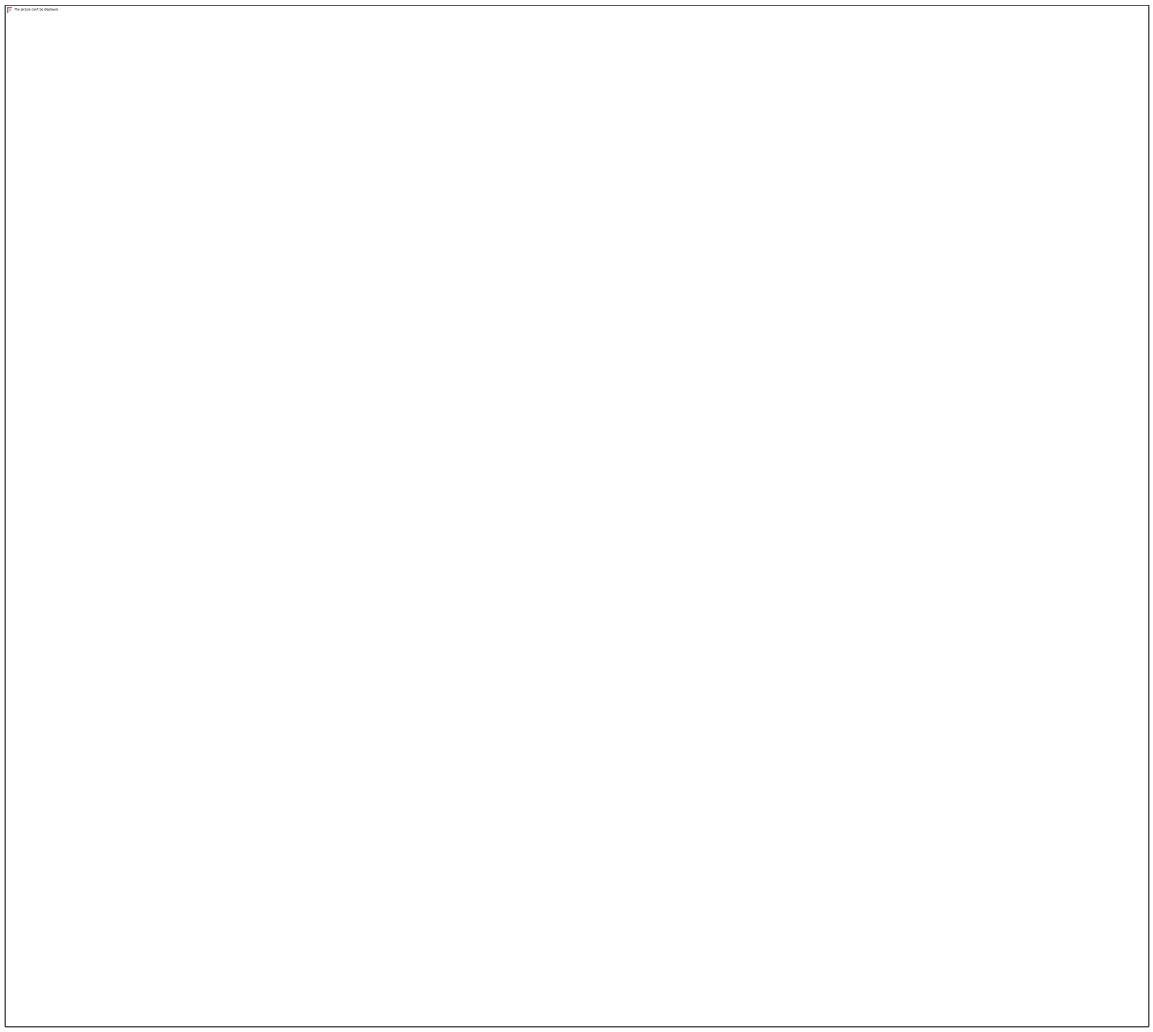
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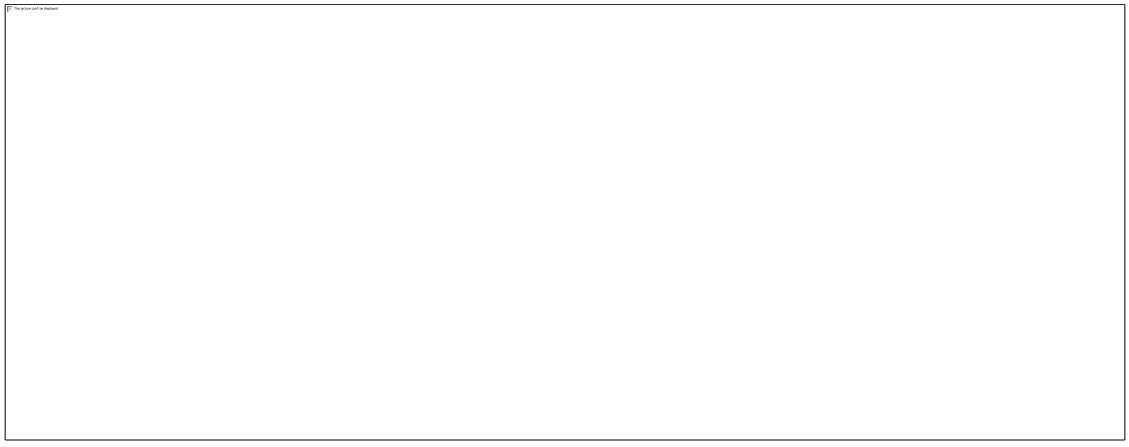
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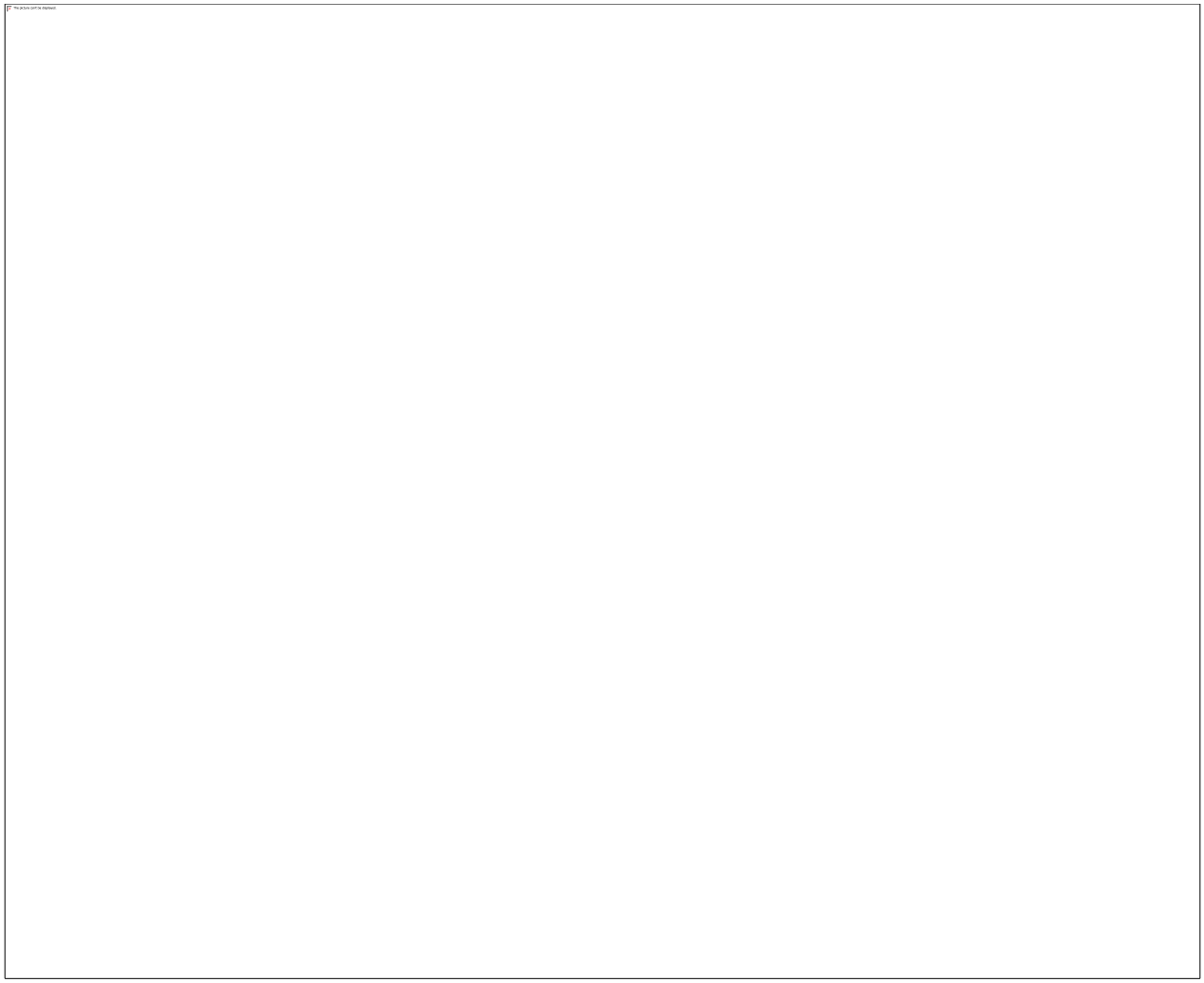


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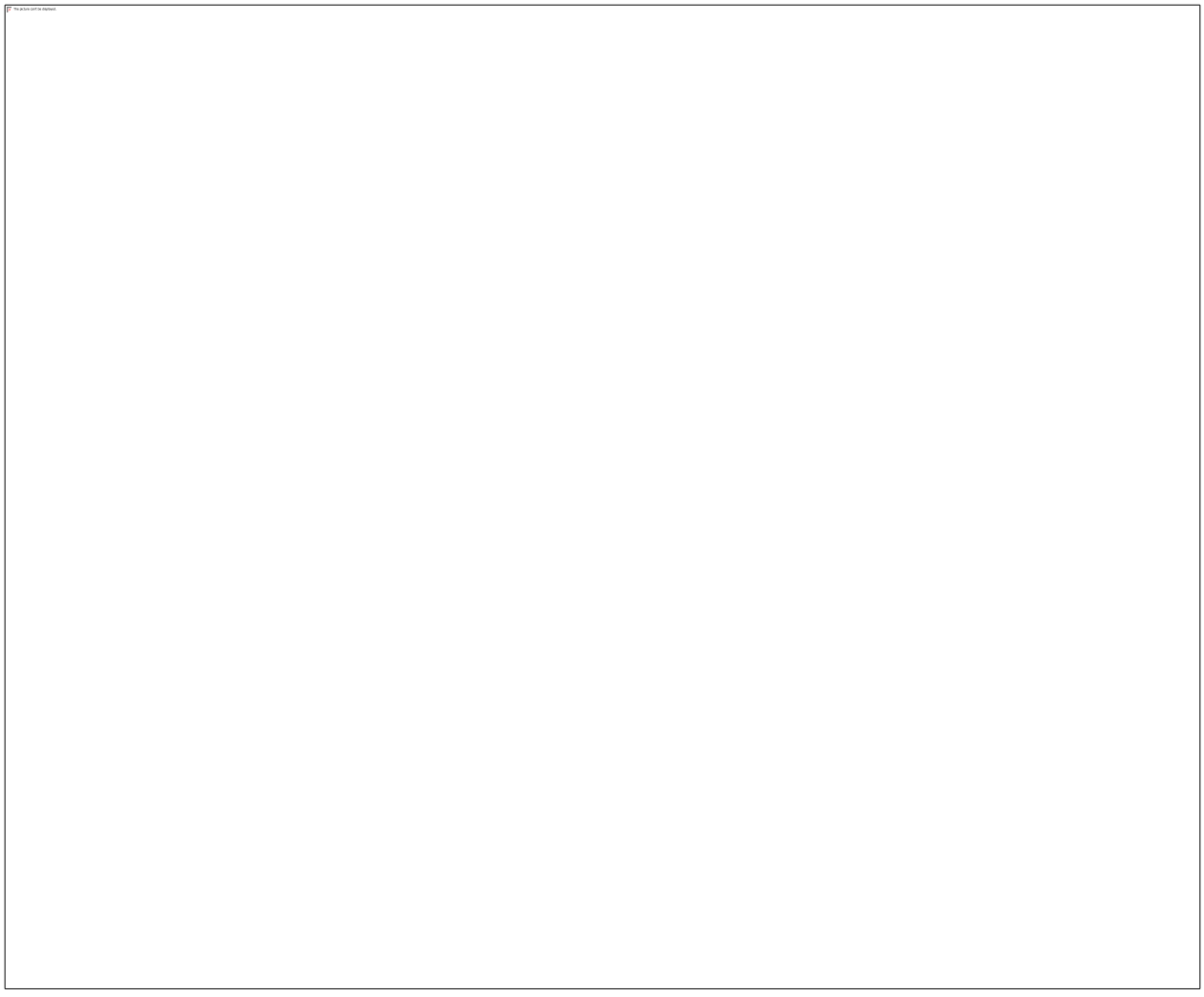
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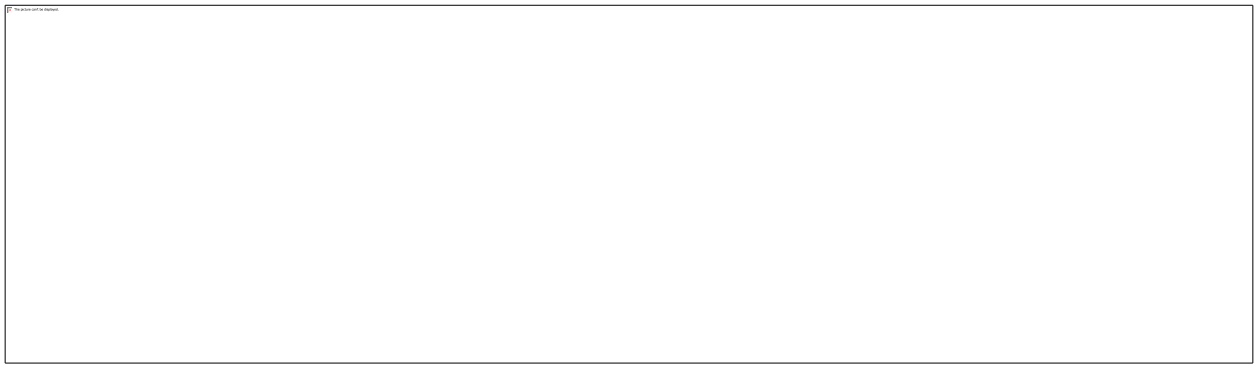

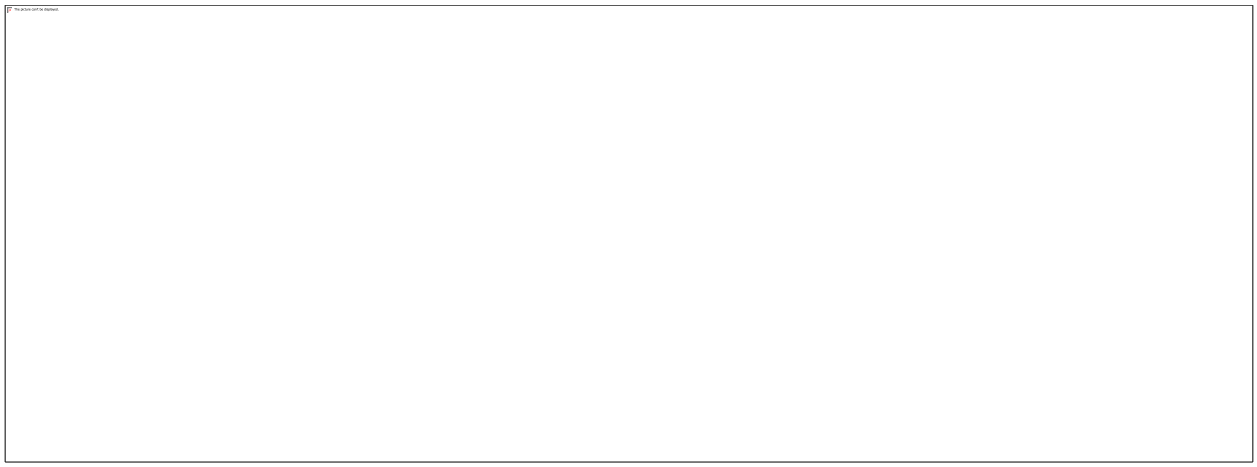

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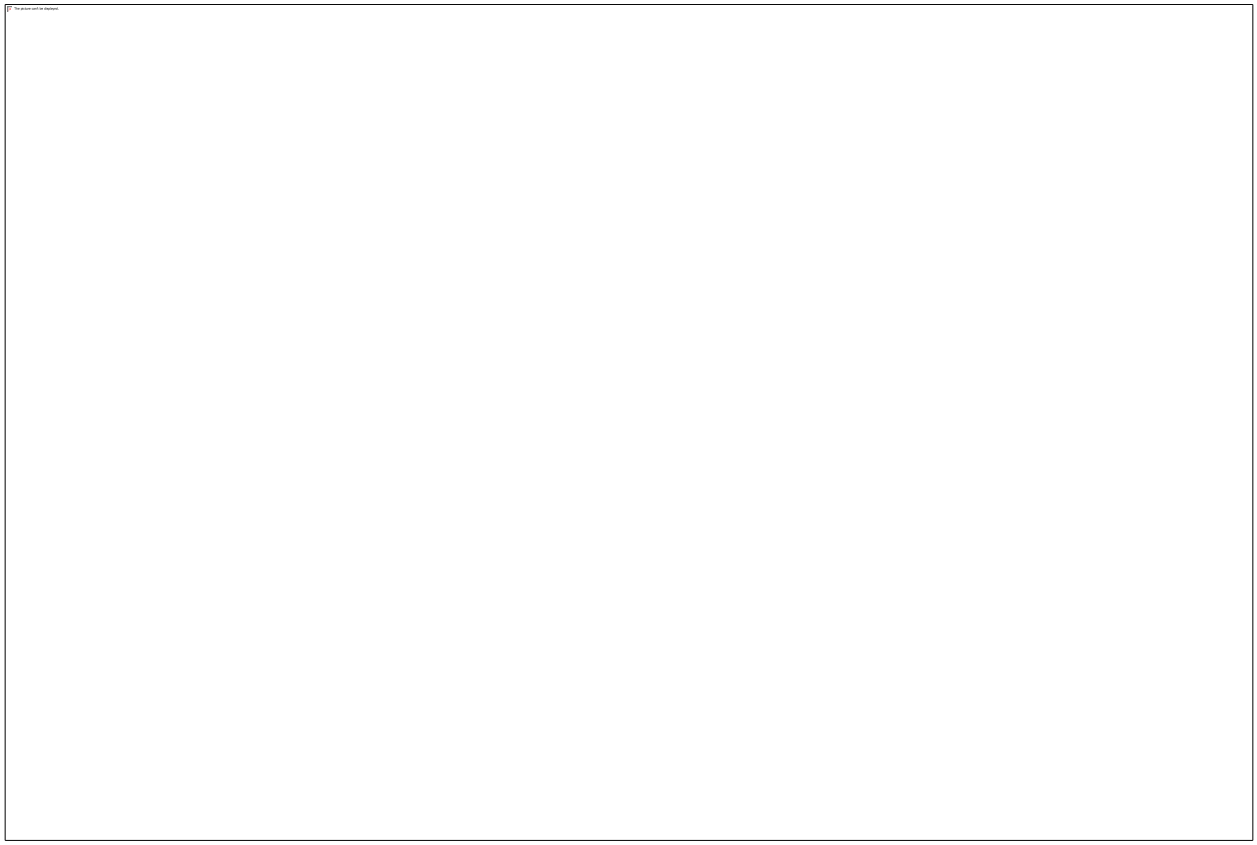


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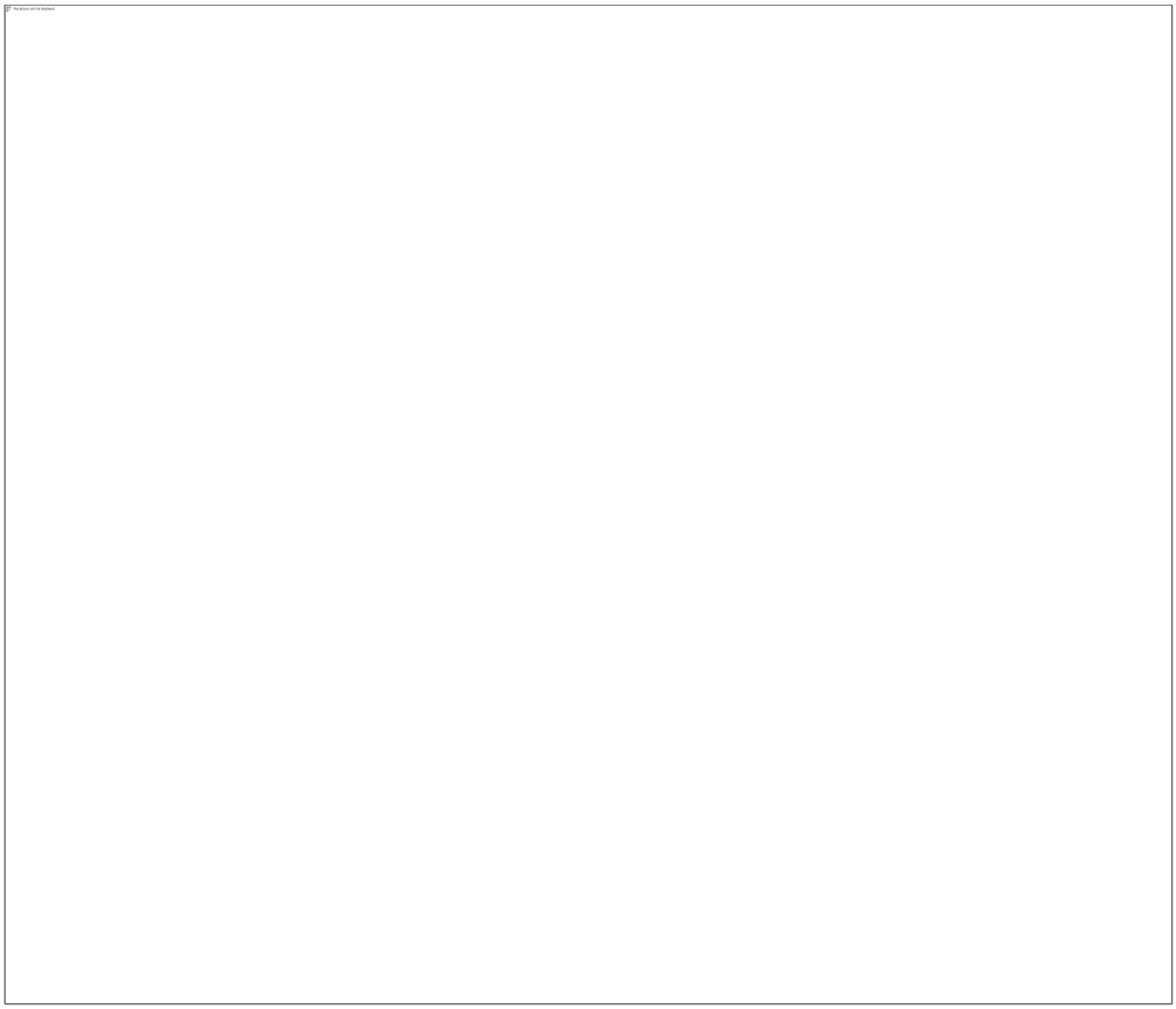
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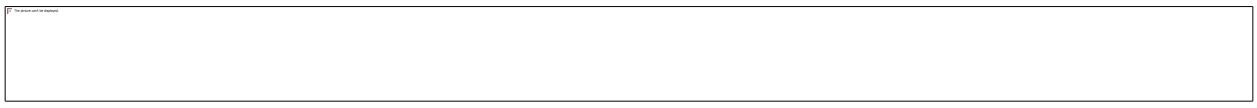


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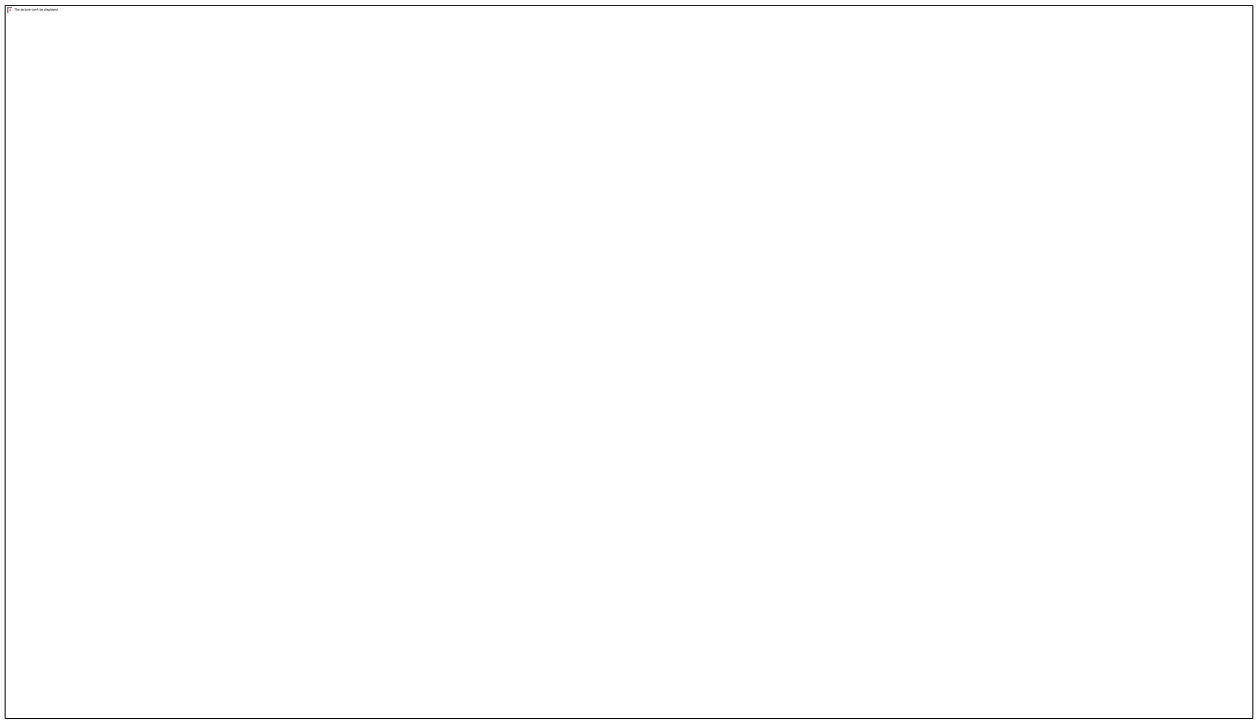
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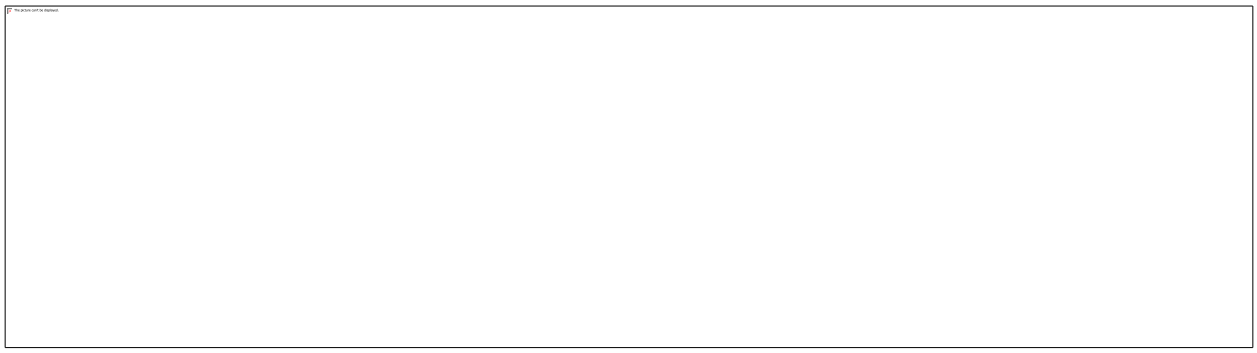
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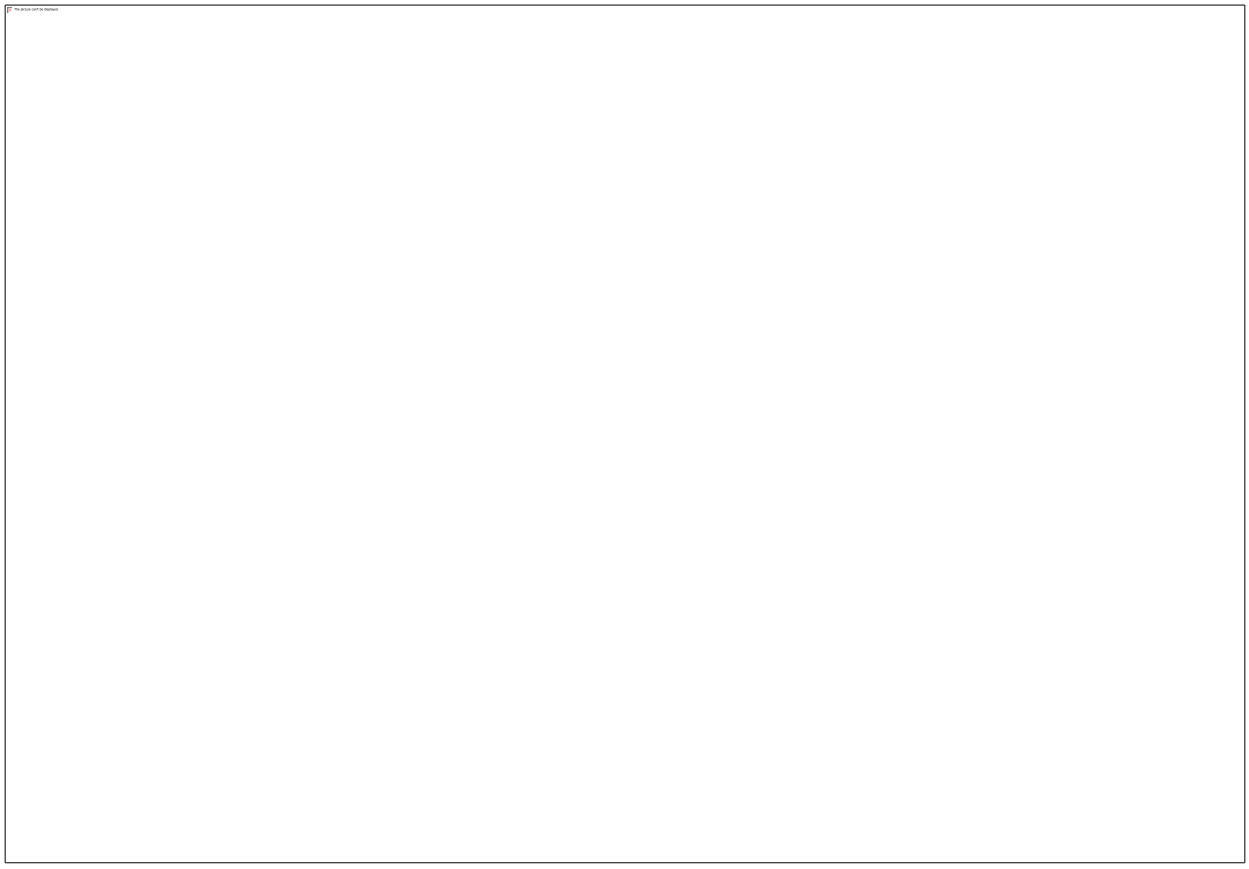

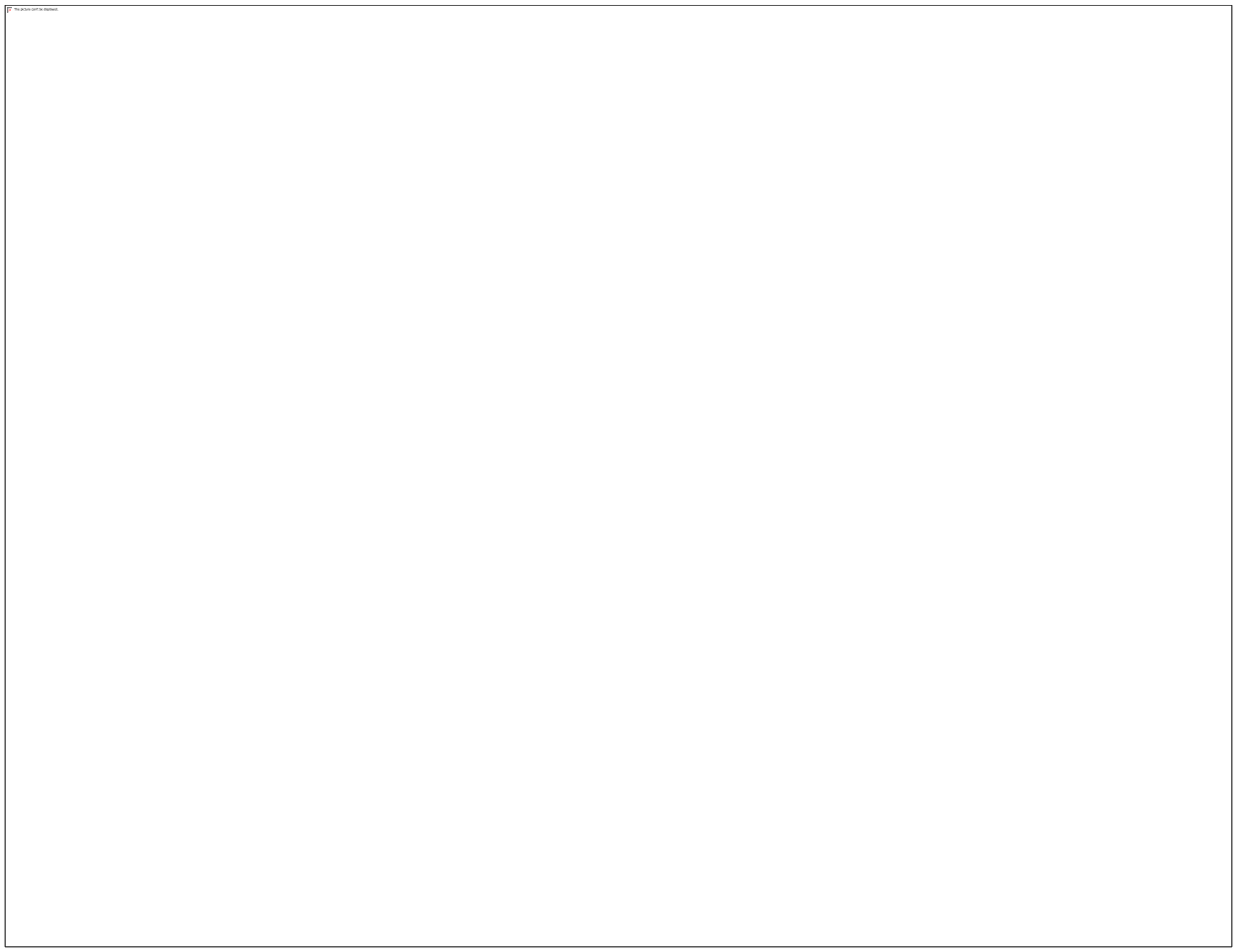
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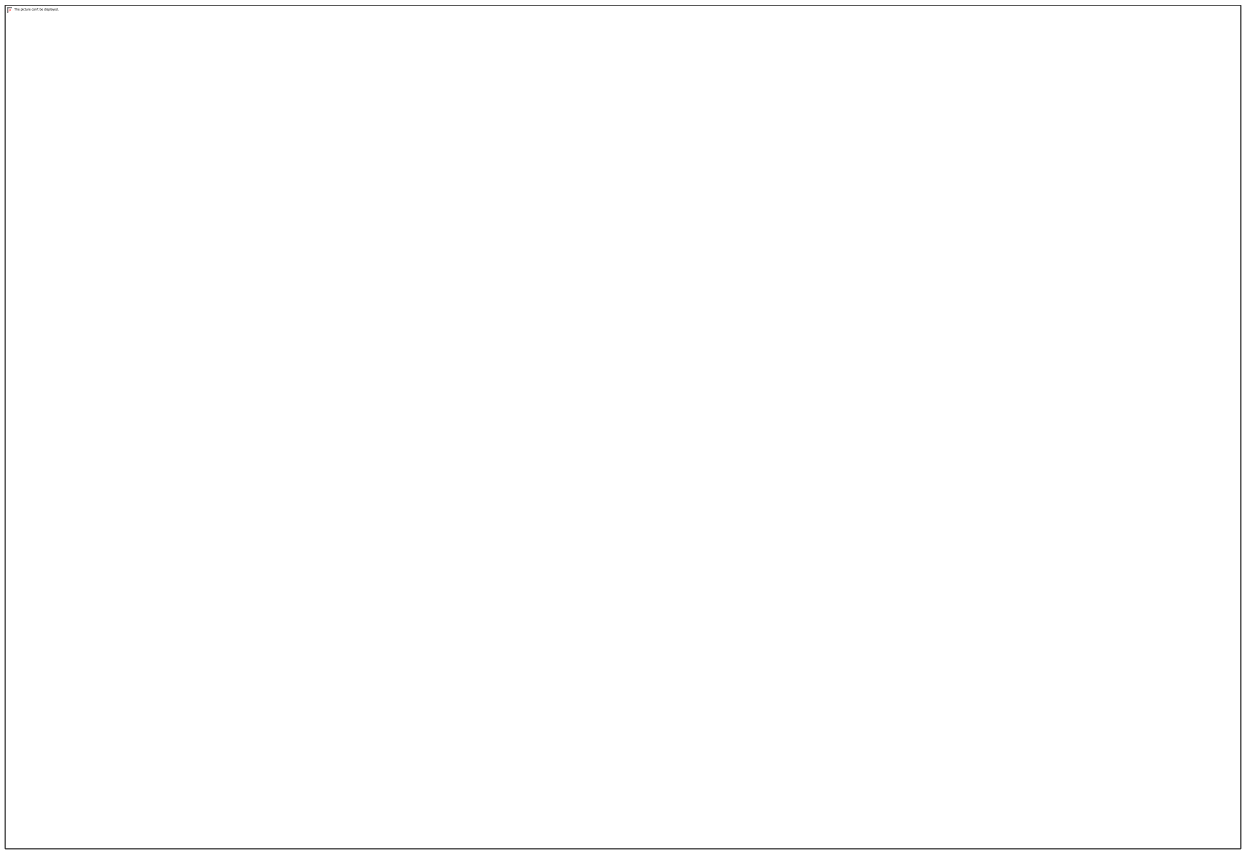

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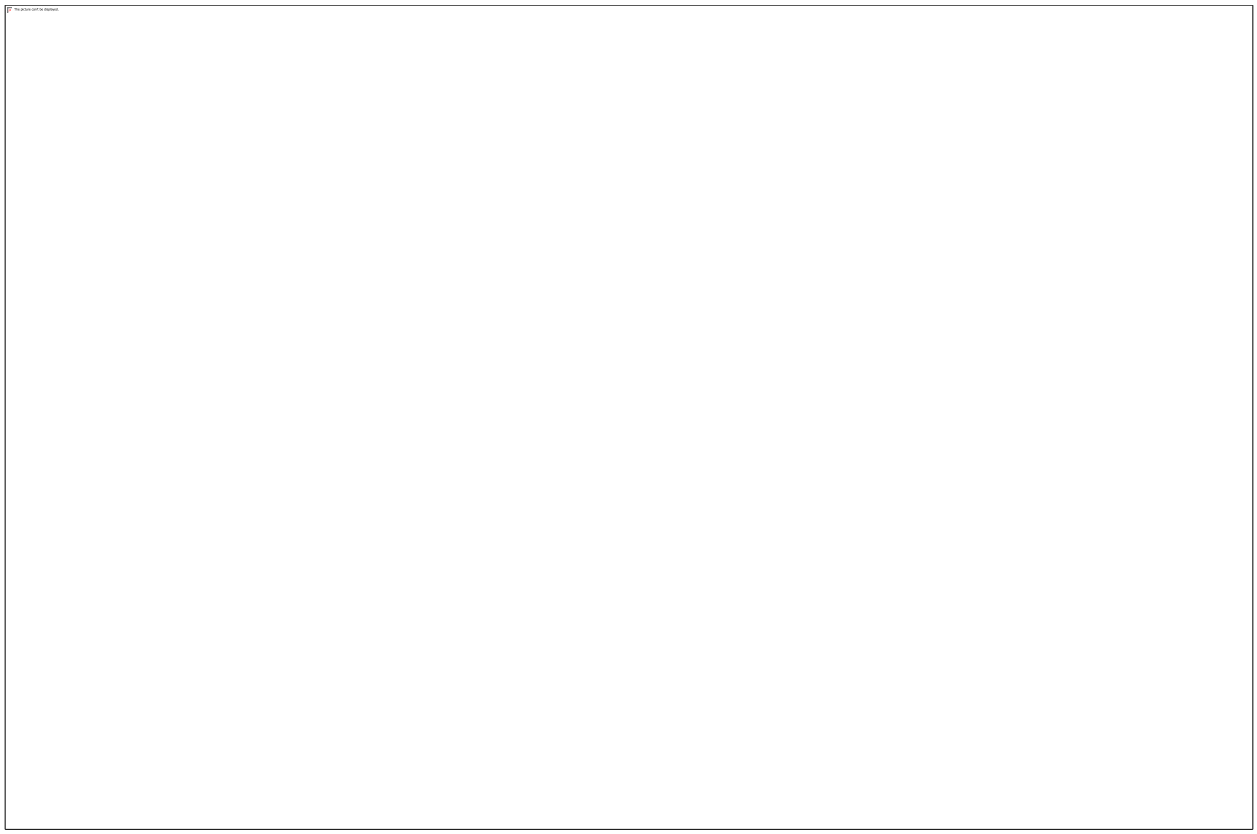
17.

Question Number	Scheme	Marks
7. (a)	$R = \sqrt{6.25}$ or 2.5	B1
	$\tan\alpha = \frac{1.5}{2} = \frac{3}{4} \Rightarrow \alpha =$ awrt 0.6435	M1A1 (3)
(b) (i)	Max Value $= 2.5$	B1$\surd$
(ii)	$\sin(\theta - 0.6435) = 1$ or $\theta -$ their $\alpha = \frac{\pi}{2}; \Rightarrow \theta =$ awrt 2.21	M1;A1$\surd$ (3)
(c)	$H_{Max} = 8.5$ (m)	B1$\surd$
	$\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 1$ or $\frac{4\pi t}{25} =$ their (b) answer $; \Rightarrow t =$ awrt 4.41	M1;A1 (3)
(d)	$\Rightarrow 6 + 2.5\sin\left(\frac{4\pi t}{25} - 0.6435\right) = 7; \Rightarrow \sin\left(\frac{4\pi t}{25} - 0.6435\right) = \frac{1}{2.5} = 0.4$	M1;M1
	$\left\{\frac{4\pi t}{25} - 0.6435\right\} = \sin^{-1}(0.4)$ or awrt 0.41	A1
	Either $t =$ awrt 2.1 or awrt 6.7	A1
	So, $\left\{\frac{4\pi t}{25} - 0.6435\right\} = \{\pi - 0.411517...$ or $2.730076...^{c}\}$	ddM1
	Times $= \{14:06, 18:43\}$	A1 (6) **[15]**



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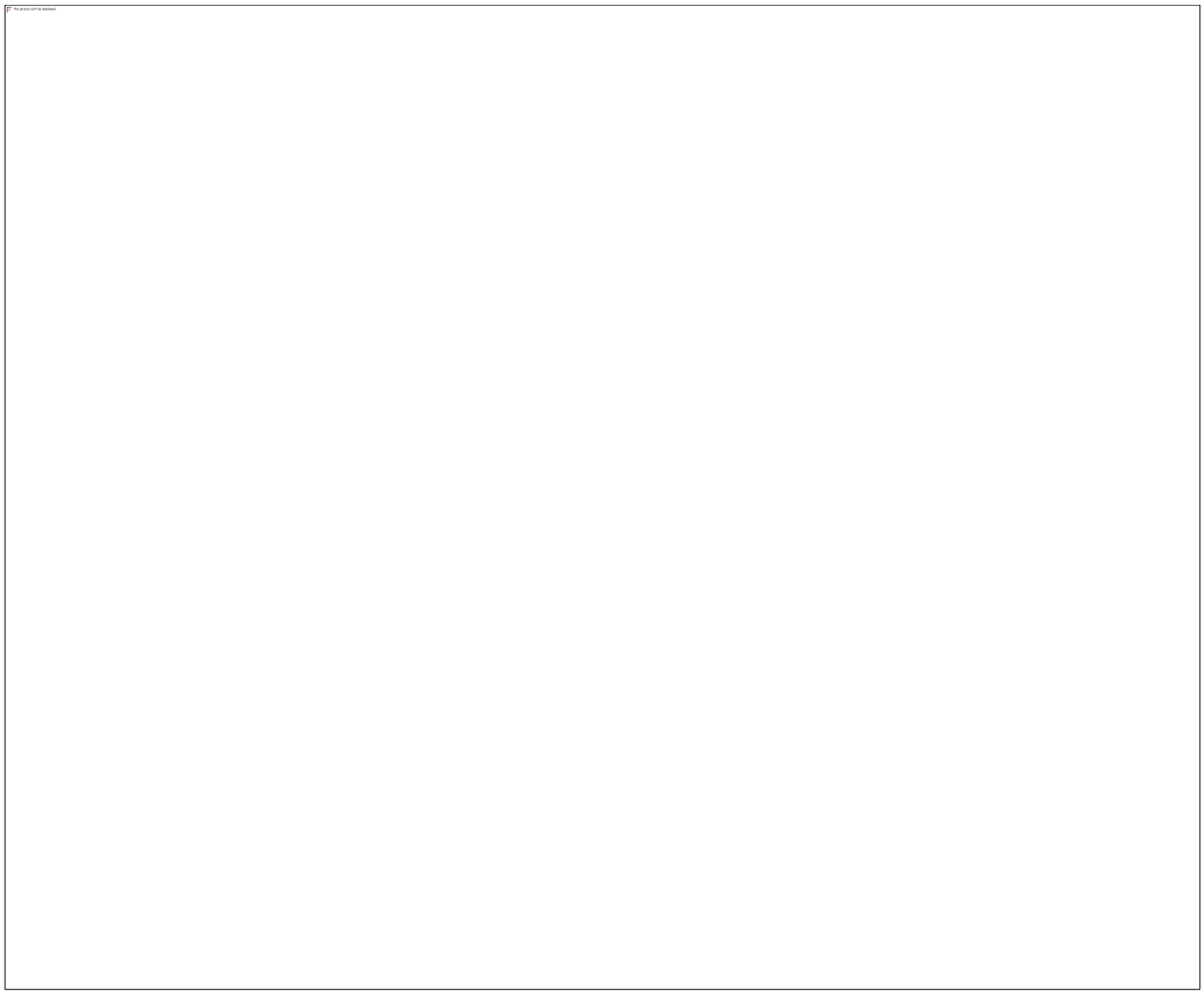
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