Trigonometric Functions - Answers

June 2017 Mathematics Advanced Paper 1: Pure Mathematics 3

Question Number	Scheme	Marks	
4.(a)	$R = \sqrt{29}$	B1	
	$R = \sqrt{29}$ $\tan \alpha = \frac{2}{5} \Rightarrow \alpha = \text{awrt } 0.381$	M1A1	
	cos2r 3		(3)
(b)	$5\cot 2x - 3\csc 2x = 2 \Rightarrow 5\frac{\cos 2x}{\sin 2x} - \frac{3}{\sin 2x} = 2$	M1	
	$\Rightarrow 5\cos 2x - 2\sin 2x = 3$	A1	(2)
(c)	$5\cos 2x - 2\sin 2x = 3 \Rightarrow \cos(2x + 0.381) = \frac{3}{\sqrt{29}}$	M1	
	$2x + 0.381 = \arccos\left(\frac{3}{\sqrt{29}}\right) \Rightarrow x = \dots$	dM1	
	x = awrt 0.30, 2.46	A1A1	
			(4)
		(9 marks)	

Alt I (c)	$5\cos 2x - 2\sin 2x = 3 \Rightarrow 10\cos^2 x - 5 - 4\sin x \cos x = 3$		
	$\Rightarrow 4 \tan^2 x + 2 \tan x - 1 = 0$	M1	
	$\Rightarrow \tan x = \frac{-1 \pm \sqrt{5}}{4} \Rightarrow x =$	dM1	
	x = awrt 0.30, 2.46	A1A1	(4)
			(+)
Alt II (c)	$5\cos 2x - 2\sin 2x = 3 \Rightarrow (5\cos 2x)^2 = (3 + 2\sin 2x)^2$ & $\cos^2 2x = 1 - \sin^2 2x$		
	$\Rightarrow 29\sin^2 2x + 12\sin 2x - 16 = 0$	M1	
	$\Rightarrow \sin 2x = \frac{-12 \pm \sqrt{2000}}{58} \Rightarrow 2x = \Rightarrow x =$	dM1	
	x = awrt 0.30, 2.46	A1A1	
			(4)

(a)

B1
$$R = \sqrt{29}$$

Condone $R = \pm \sqrt{29}$ (Do not allow decimals for this mark Eg 5.39 but remember to isw after $\sqrt{29}$)

M1
$$\tan \alpha = \pm \frac{2}{5}$$
, $\tan \alpha = \pm \frac{5}{2} \Rightarrow \alpha = ...$

If R is used to find α accept $\sin \alpha = \pm \frac{2}{R}$ or $\cos \alpha = \pm \frac{5}{R} \Rightarrow \alpha = ...$

A1 $\alpha = \text{awrt } 0.381$

Note that the degree equivalent $\alpha = \text{awrt } 21.8^{\circ}$ is A0

(b)

M1 Replaces
$$\cot 2x$$
 by $\frac{\cos 2x}{\sin 2x}$ and $\csc 2x$ by $\frac{1}{\sin 2x}$ in the lhs

Do not be concerned by the coefficients 5 and -3.

Replacing $\cot 2x$ by $\frac{1}{\tan 2x}$ does not score marks until the $\tan 2x$ has been replaced by $\frac{\sin 2x}{\cos 2x}$

They may state $\times \sin 2x \Rightarrow 5\cos 2x - 3 = 2\sin 2x$ which implies this mark

A1 cso $5\cos 2x - 2\sin 2x = 3$ There is no need to state the value of 'c'

The notation must be correct. They cannot mix variables within their equation

Do not accept for the final A1 $\tan 2x = \frac{\sin 2x}{\cos 2x}$ within their equations

(c)

M1 Attempts to use part (a) and (b). They must be using their R and α from part (a) and their c from part (b)

Accept $\cos(2x \pm \alpha') = \frac{c'}{R'}$ Condone $\cos(\theta \pm \alpha') = \frac{c'}{R'}$ or even $\cos(x \pm \alpha') = \frac{c'}{R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 **correctly** and in that order to reach x = ...Don't be concerned if they change the variable in the question and solve for $\theta =$ (as long as all operations have

been undone). You may not see any working. It is implied by one correct answer. You may need to check with a calculator.

Eg for an incorrect
$$\alpha \cos(2x+1.19) = \frac{3}{\sqrt{29}} \Rightarrow x = -0.105$$
 would score M1 dM1 A0 A0

- Al One solution correct, usually x = 0.3/0.30 or x = 2.46 or in degrees 17.2° or $141.(0)^{\circ}$
- Al Both solutions correct awrt x = awrt 0.30, 2.46 and no extra values in the range.

Condone candidates who write 0.3 and 2.46 without any (more accurate) answers

In degrees accept awrt 1 dp 17.2°,141.(0)° and no extra values in the range.

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Special case: For candidates who are misreading the question and using their part (a) with 2 on the rhs.

They will be allowed to score a maximum of SC M1 dM1 A0 A0

M1 Attempts to use part (a) with 2. They must be using their R and α from part (a)

Accept
$$\cos(2x \pm \alpha') = \frac{2}{R'}$$
 Condone $\cos(\theta \pm \alpha') = \frac{2}{R'}$ or even $\cos(x \pm \alpha') = \frac{2}{R'}$ for the first M

dM1 Score for dealing with the cos, the α and the 2 correctly and in that order to reach x = ...

You may not see any working. It is implied by one correct answer. You may need to check with a calculator.

Eg for an correct
$$\alpha$$
 and $R \cos(2x + 0.381) = \frac{2}{\sqrt{29}} \Rightarrow x = 0.405$

.....

Alt to part (c)

- M1 Attempts both double angle formulae condoning sign slips on $\cos 2x$, divides by $\cos^2 x$ and forms a quadratic in tan by using the identity $\pm 1 \pm \tan^2 x = \sec^2 x$
- dM1 Attempts to solve their quadratic in tanx leading to a solution for x.
- A1 A1 As above

June 2016 Mathematics Advanced Paper 1: Pure Mathematics 3

Question	Scheme	Marks
3.(a)	$R = \sqrt{5}$	B1
	$\tan \alpha = \frac{1}{2} \Rightarrow \alpha = 26.57^{\circ}$	M1A1
		(3)
(b)	$\frac{2}{2\cos\theta - \sin\theta - 1} = 15 \Rightarrow \frac{2}{\sqrt{5}\cos(\theta + 26.6^\circ) - 1} = 15$	
	$\Rightarrow \cos(\theta + 26.6^{\circ}) = \frac{17}{15\sqrt{5}} = (awrt\ 0.507)$	M1A1
	$\theta + 26.57^{\circ} = 59.54^{\circ}$	
	$\Rightarrow \theta = awrt 33.0^{\circ} \text{ or } awrt 273.9^{\circ}$	A1
	$\theta + 26.6^{\circ} = 360^{\circ} - \text{their'} 59.5^{\circ}$	dM1
	$\Rightarrow \theta = awrt \ 273.9^{\circ} \text{ and } awrt \ 33.0^{\circ}$	A1
		(5)
(c)	θ – their 26.57° = their 59.54° $\Rightarrow \theta =$	M1
` '	$\theta = \text{awrt } 86.1^{\circ}$	A1
		(2)
		(10 marks)

(a)

B1 $R = \sqrt{5}$. Condone $R = \pm \sqrt{5}$ Ignore decimals

M1
$$\tan \alpha = \pm \frac{1}{2}$$
, $\tan \alpha = \pm \frac{2}{1} \Rightarrow \alpha = ...$

If their value of R is used to find the value of α only accept $\cos \alpha = \pm \frac{2}{R}$ OR $\sin \alpha = \pm \frac{1}{R} \Rightarrow \alpha = ...$

A1 $\alpha = \text{awrt } 26.57^{\circ}$

(b)

M1 Attempts to use part (a) $\Rightarrow \cos(\theta \pm \text{their } 26.6^{\circ}) = K$, |K|, 1

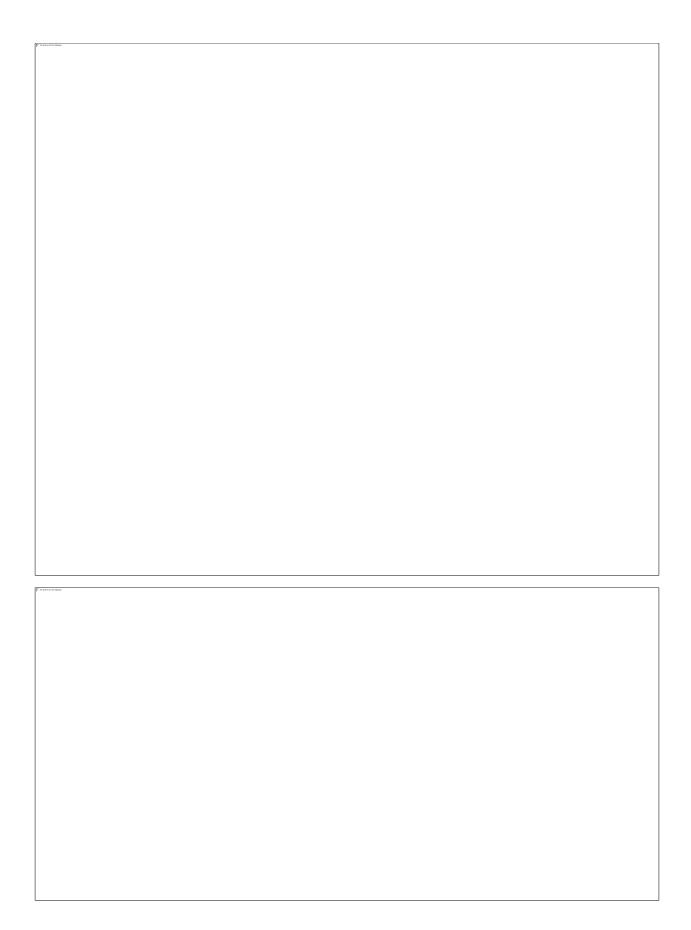
A1
$$\cos(\theta \pm \text{their } 26.6^{\circ}) = \frac{17}{15\sqrt{5}} = (\text{awrt } 0.507)$$
. Can be implied by $(\theta \pm \text{their } 26.6^{\circ}) = \text{awrt } 59.5^{\circ} / 59.6^{\circ}$

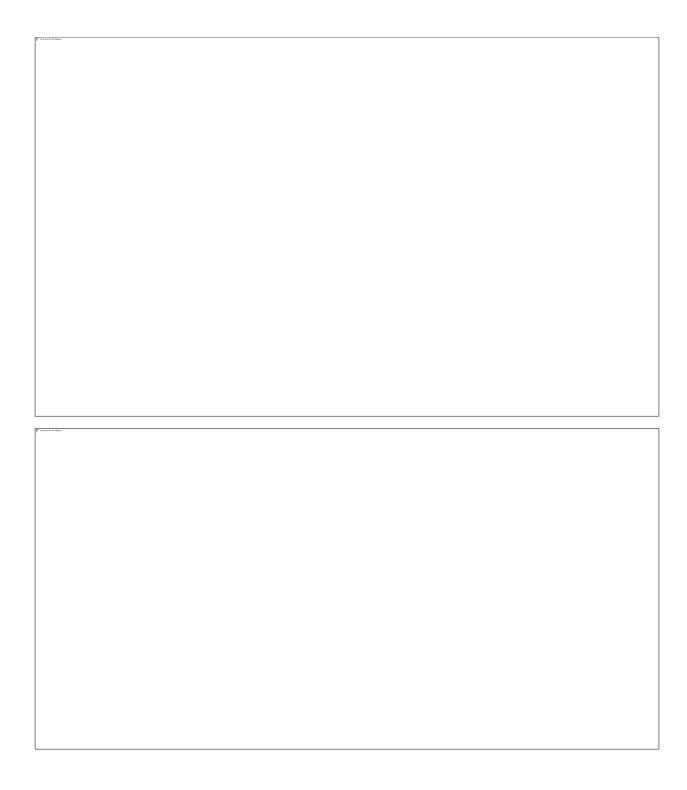
A1 One solution correct, $\theta = awrt 33.0^{\circ}$ or $\theta = awrt 273.9^{\circ}$ Do not accept 33 for 33.0.

dM1 Obtains a second solution in the range. It is dependent upon having scored the previous M. Usually for $\theta \pm$ their $26.6^{\circ} = 360^{\circ} -$ their $59.5^{\circ} \Rightarrow \theta = ...$

A1 Both solutions $\theta = awrt 33.0^{\circ}$ and $awrt 273.9^{\circ}$. Do not accept 33 for 33.0. Extra solutions inside the range withhold this A1. Ignore solutions outside the range $0 = \theta < 360^{\circ}$







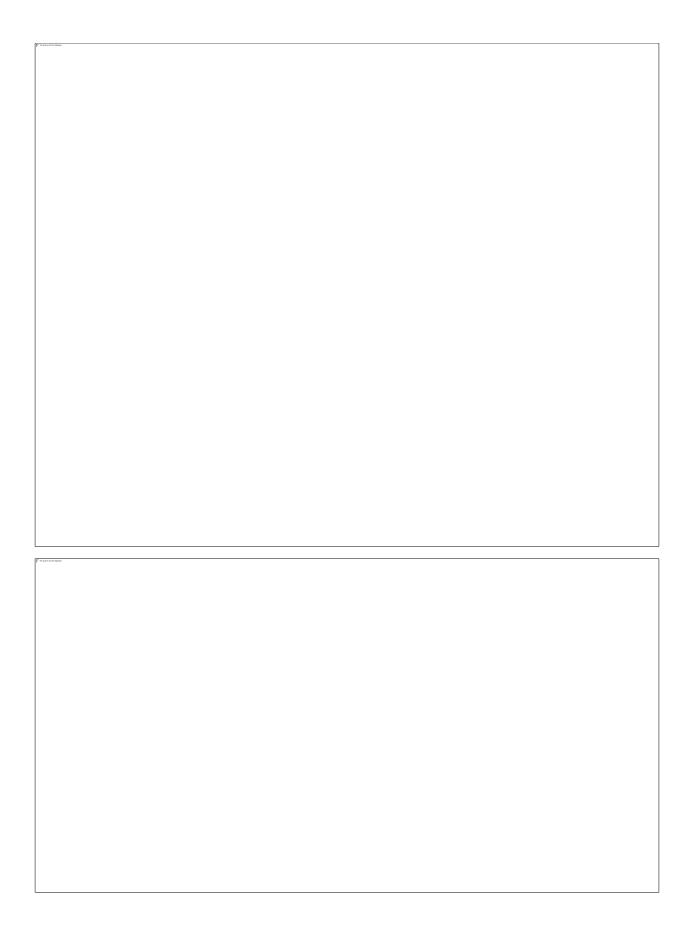
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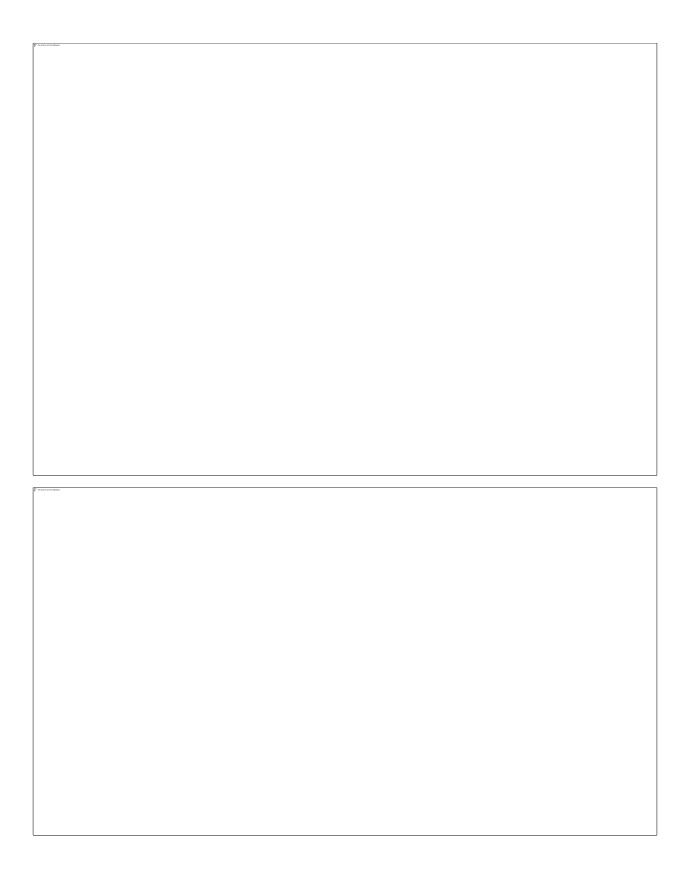
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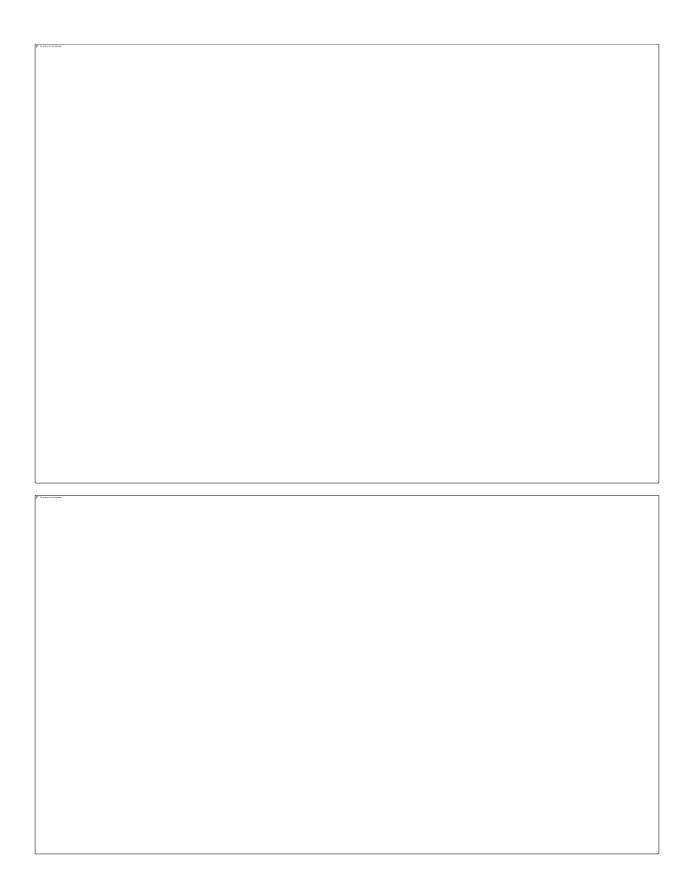
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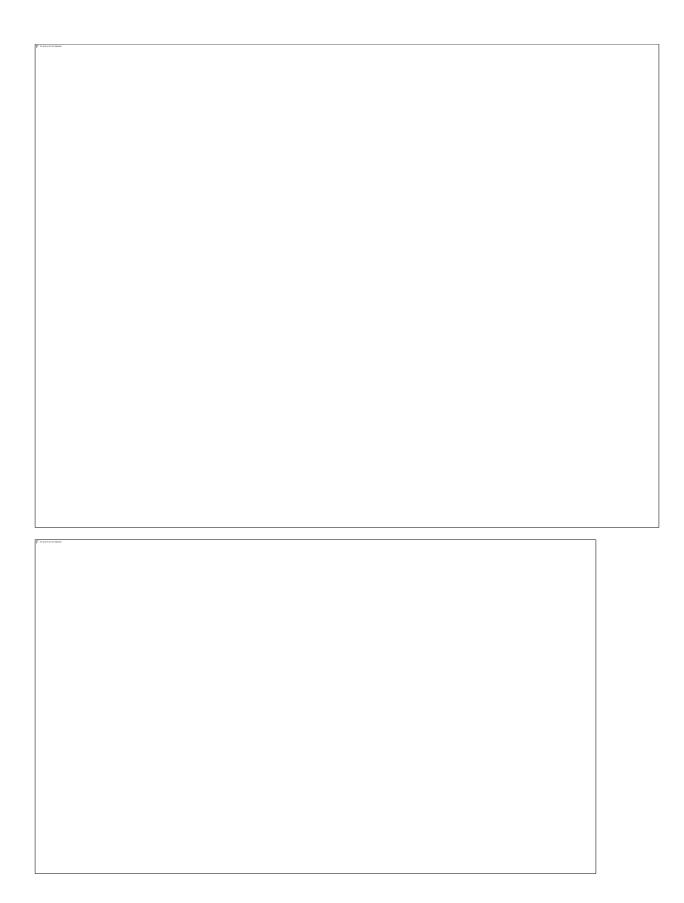
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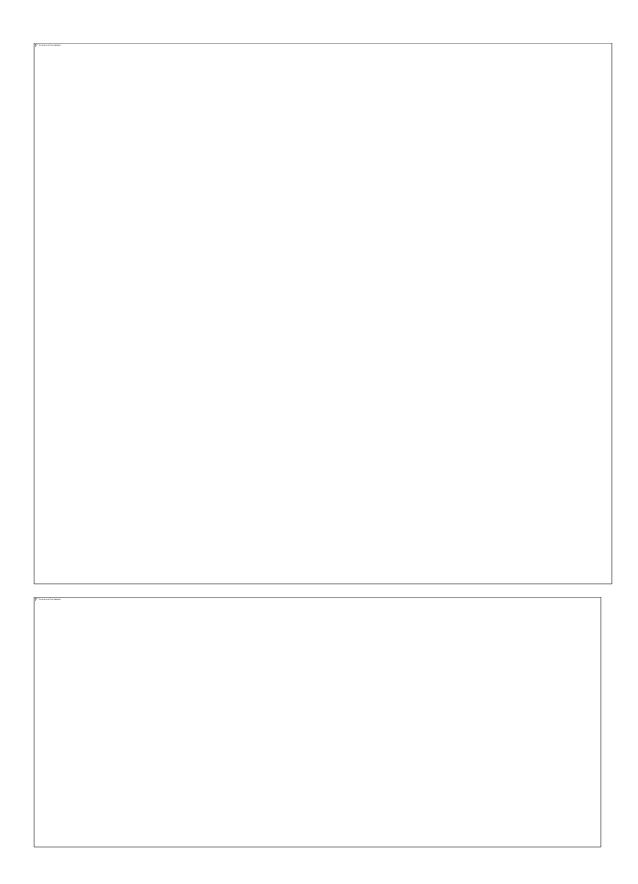






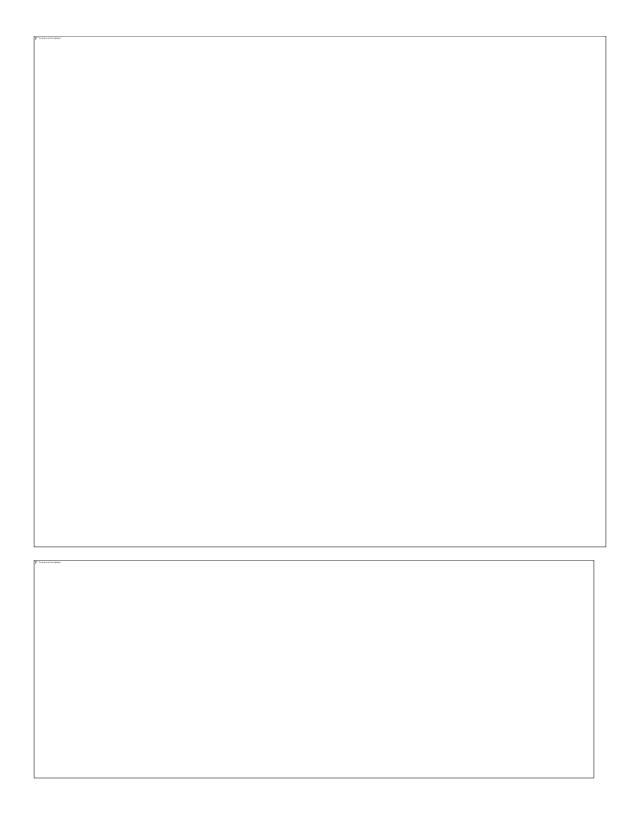
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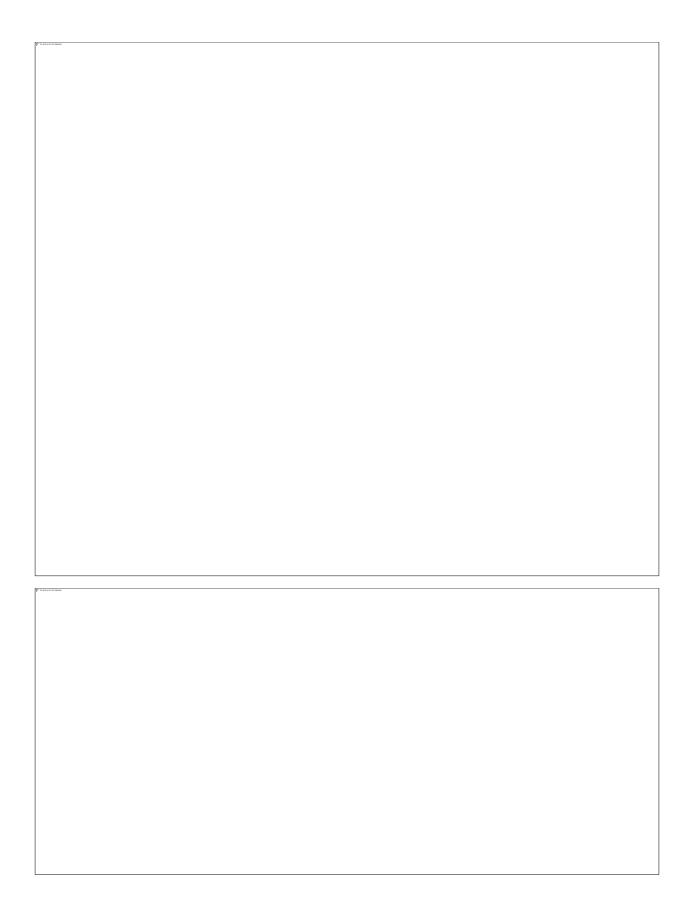


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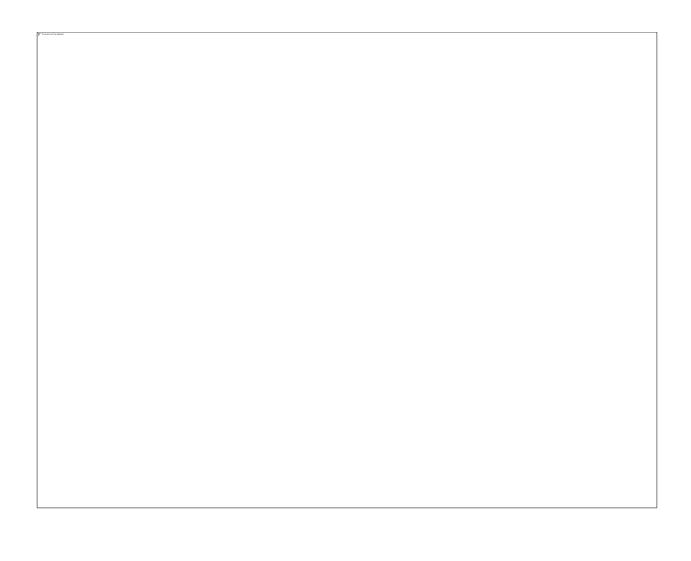


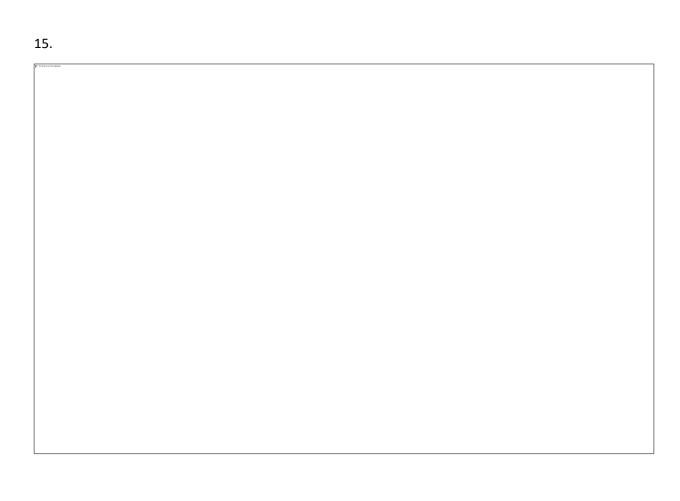
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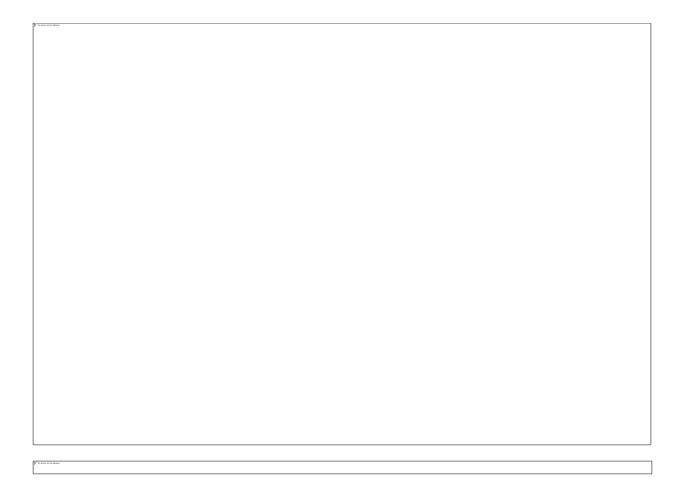
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